OFDM tutorial, part 1: PAPR, ECC, and DAR

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Part two shows how to enhance OFDM with MIMO and other techniques. The many papers noted in this series are listed in the references section.

This chapter covers specific wireless standards and air interfaces. For wireless systems currently actively under development there are two important classes: those based on OFDM, including WLAN, standards and proposed for future high speed mobile systems, and those based on CDMA, especially 3rd generation mobile standards. For this reason the chapter is divided into two sections: Section 2.2 covers OFDM-based systems, while Section 2.3 covers CDMA. Multi Carrier–CDMA (MC-CDMA) systems, which can be regarded as a combination of the two, are discussed in Subsection 2.3.4.

OFDM systems

During the past 15 years, Orthogonal Frequency Division Multiplexing (OFDM) has been gaining year after year a well-deserved reputation, demonstrating its high data rate and robustness to wireless environments capabilities. In the multipath environment, broadband communications will suffer from frequency selective fading. In such a situation, the above technologies do not work optimally and a transport scheme better suited for the environment is needed. For this reason, OFDM became very popular recently [Enge02].

OFDM is an attractive modulation scheme used in broadband wireless systems that encounter large delay spreads. The complexity of Maximum Likelihood (ML) detection or even sub-optimal equalization schemes needed for single carrier modulation grows exponentially with the bandwidth delay spread product. OFDM avoids temporal equalization altogether, using a cyclic prefix technique with a small penalty in channel capacity [PaNG03].

Where Line-of-Sight (LoS) cannot be achieved, there is likely to be significant multipath dispersion, which could limit the maximum data rate. Technologies like OFDM are probably best placed to overcome these, allowing nearly arbitrary data rates on dispersive channels. OFDM, in particular for broadband systems in dispersive environments, is a technology that could have a place in the 4G
Although the principle of OFDM communication has been around for several decades, it was only in the last decade that it started to be used in commercial systems. The most important wireless applications that make use of OFDM are Digital Audio Broadcast (DAB), DVB, WLAN and more recently Wireless Local Loop (WLL) [Enge02].

The DAB system is seen as the future of radio as it makes more efficient use of crowded airwaves and provides CD quality sound that is noticeably better than an FM analogue broadcast. DAB makes use of an OFDM transmission scheme with differential Quaternary PSK (QPSK) modulation. The DVB system is very similar to DAB standard, but is intended for broadcasting of digital television signals. Due to the high data rates, the DVB system uses an 8 MHz bandwidth. The subcarriers in the OFDM signal are modulated with a higher order Quadrature Amplitude Modulation (QAM) constellation, with up to 64 points.

The third generation of WLAN systems is intended to offer high data rates in the 5 GHz frequency band and more recently in the 2.5 GHz band. The communication is based on OFDM in the 20 MHz bandwidth. Per subcarrier, the modulation schemes are Binary Phase Shift Keying (BPSK), QPSK, 16-QAM and 64-QAM. Together with a variable error coding rate, this allows the data rates to be adapted from 6 Mbit/s to 54 Mbit/s depending on the propagation environment. Future WLAN standards are being studied to overcome that range.

Wireless Local Loops provide high speed Internet access and multimedia services to fixed users. WLL is a competitive technology to Very-High-Rate Digital Subscriber Line (VDSL) and cable modems. OFDM is one of the supported transport schemes for WLL technologies [Enge02].

The simplification of equalization and the high bandwidth efficiency and flexibility are the main motivations for using OFDM, which is always a preferred alternative if a high data rate is to be transmitted over a multipath channel with large maximum delay [Corr01].

Channel coding plays an important role in OFDM systems. Due to the narrowband subcarriers and the appropriate cyclic prefix, OFDM systems suffer from flat fading. In this situation, efficient channel coding leads to a very high coding gain, especially if soft decision decoding is applied. For this reason OFDM systems will always have to make use of channel coding [AlLa87]. Furthermore, OFDM allows multiple access techniques to certain time and/or frequency regions of the channel in a very simple way [RoGr97].

Finally, OFDM signals have a large peak-to-mean power ratio due to the superposition of all subcarrier signals, therefore in each Transceiver (TRX), the power amplifier will limit the OFDM signal by its maximal output power. This also disturbs the orthogonality between subcarriers, leading to both intercarrier and out-of-band interferences, which is unacceptable [Corr01].

**Subcarriers**

An OFDM signal consists of \( N \) subcarriers spaced by the frequency distance \( \Delta f \), thus, the total system bandwidth \( B \) is divided into \( N \) equidistant subchannels. On each subcarrier, the symbol duration \( T_s = 1/\Delta f \) is \( N \) times as large as in the case of a single carrier transmission system covering the same bandwidth. Additionally, each subcarrier signal is extended by a guard interval – called cyclic prefix – with the length \( T_g \). All subcarriers are mutually orthogonal within the symbol duration \( T_s \). The \( k \)th subcarrier signal is described analytically by the function \( g_k(t) \), \( k = 0, \ldots, N - 1 \). For each subcarrier a rectangular pulse shaping is applied.
The guard interval is added to the subcarrier signal in order to avoid Inter-Symbol Interference (ISI), which occurs in multipath channels. At each Receiver (RX), the cyclic prefix is removed and only the time interval \([0, T_s]\) is evaluated. The total OFDM block duration is \(T = T_s + T_g\). Each subcarrier can be modulated independently with the complex modulation symbol \(s_{n,k}\), where the subscript \(n\) refers to time and \(k\) to the subcarrier index inside the considered OFDM block. Thus, within the symbol duration \(T\) the following signal of the \(n\)th OFDM block is formed:

\[
g_k(t) = \begin{cases} \ e^{j2\pi k \Delta f t} & \forall t \in [-T_g, T_s] \\ 0 & \forall t \notin [-T_g, T_s] \end{cases} \quad (2.1)
\]

Due to the rectangular pulse shaping of the signal, the spectra of the subcarriers are \(\text{sinc}\) functions, Figure 2-1.

\[
s_n(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s_{n,k} g_k(t - nT) \quad (2.2)
\]

The spectra of the subcarriers overlap, but the subcarrier signals are mutually orthogonal, and the modulation symbol \(s_{n,k}\) can be recovered by a simple correlation:

\[
s_{n,k} = \frac{\sqrt{N}}{T_s} \left( s_n(t), g_k(t - nT) \right) \quad (2.3)
\]

In a practical application, the OFDM signal \(s_n(t)\) is generated in a first step as a discrete-time signal in the digital signal processing part of the TRX. As the bandwidth of an OFDM system is \(B = N\Delta f\), the signal must be sampled with sampling time \(t = 1/B = 1/N\Delta f\). The samples of the signal are written as \(s_{n,i}, i = 0, 1, ..., N - 1\), and can be calculated by an Inverse DFT (IDFT) which is typically implemented as an Inverse FFT (IFFT).
The subcarrier orthogonality is not affected at the output of a frequency selective radio channel; therefore, the received signal $r_n(t)$ can be separated into the orthogonal subcarrier signals by a correlation technique according to (2.3). Alternatively, the correlation at RX can be done as a Discrete Fourier Transform (DFT) or a Fast Fourier Transform (FFT) respectively:

$$R_{n,k} = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} r_{n,i} e^{-j \frac{2\pi ik}{N}}$$  \hspace{1cm} (2.5)$$

where $r_{n,i}(t)$ is the $i$th sample of the received signal $r_n(t)$, and $R_{n,k}$ is the recovered complex symbol of the $k$th subcarrier. If the subcarrier spacing $\Delta f$ is chosen to be much smaller than the coherence bandwidth, and the symbol duration $T$ much smaller than the coherence time of the channel, then the transfer function of the radio channel $H(f, t)$ can be considered constant within the bandwidth $\Delta f$ of each subcarrier and the duration of each modulation symbol $S_{n,k}$. In this case, the effect of the radio channel is only a multiplication of each subcarrier signal $g_k(t)$ by a gain factor $H_{n,k} = H(k\Delta f, nT)$.

As a result, the received complex symbol $R_{n,k}$ after the FFT is

$$R_{n,k} = H_{n,k} . S_{n,k} + N_{n,k}$$  \hspace{1cm} (2.6)$$

with $N_{n,k}$ being Additive White Gaussian Noise (AWGN).

**OFDM: a very dynamic signal**

OFDM is an attractive modulation for frequency selective channels since it allows a very simple mitigation of ISI using a guard interval. However, a well-known drawback of OFDM is that transmitted signals exhibit a Gaussian-like time domain waveform with some relatively high peaks.

This results in many difficulties to guarantee the linear behavior of the system over its large dynamic range. The common measure used to characterize the effects of non-linearities in the OFDM signal is the Peak to Average Power Ratio (PAPR). However, this measure is not necessarily the most representative parameter to report on non-linear effects on OFDM signals.

**Mitigating the PAPR**

Many studies have been conducted on how to reduce the OFDM signal PAPR. Figure 2-2 shows the shape of the probability of the PAPR in common OFDM systems.
The most classical way is to generate an OFDM signal with low PAPR. For this, coding solutions [WiJo95], [Nee96] and phase shifting [Frie97], [BaFH96], [MuHu97b], [MuHu97a], [Tell98] have been extensively researched.

Another solution is to clip intentionally the amplitude of the transmitted signal leading to nonlinear distortion [LiCi98], [NeWi98], [O'Lo94], [DiWu98], [WuGo99], which cannot be efficiently corrected with a classical linear receiver, even using an Error Correcting Code (ECC) [TeHC03].

Kim and Stuber [KiSt99] propose an iterative non-linear decoder that corrects the clipping effect on OFDM transmissions, called Decision Aided Reconstruction (DAR). An algorithm inspired from this previous study and taking advantage of the error correcting code present in high rate Coded-OFDM (COFDM) transmissions in a turbo fashion was investigated [GeCD04].

As described on Figure 2-3, a convolutionally binary sequence is mapped onto QAM symbols \( \mathbf{X} \) and modulated using an IFFT in an \( N \)-point output sequence, such as:

\[
    x_m = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{\frac{2j\pi mk}{N}}, \quad 0 \leq m \leq N - 1 \quad (2.7)
\]

where \( \mathbf{X} = \{X_k\}_{k=0}^{N-1} \) is the transmitted symbol sequence, and \( N \) is the OFDM block size.

A guard interval is added to the \( \mathbf{X} \) sequence as:

\[
    x_k^G = x_{k+N-G}, \quad 0 \leq k \leq N + G - 1 \quad (2.8)
\]

where \( G \) is the length of the cyclic prefix, and \((k)\) is the residue of \( k \) modulo \( N \). Finally the clipping
operation is performed on the time-sequence $x^c$ as:

$$x^c_k = \begin{cases} \frac{x^G_k}{A e^{(a r g x^G_k)}} & |x^G_k| \leq A, \quad 0 \leq k \leq N + G - 1 \\ |x^G_k| > A & \end{cases}$$

(2.9)

where $x^c$ is the clipped output sequence and $A$ is the clipping amplitude. The Clipping Ratio (CRa) is defined as:

$$C Ra[dB] = 20 \log \frac{A}{\sigma_x} dB$$

(2.10)

where $\sigma_x$ is the standard deviation of $x_i$. After transmission through the channel and removal of the cyclic prefix, the signal can be written as:

$$y_k = \sum_{m=0}^{M} h_m x^c_{k-m} + n_k, \quad 0 \leq k \leq N - 1$$

(2.11)

where $h_m$ is the channel coefficient at the lag $m$ and $n_k$ is a zero-mean AWGN with variance $N_0$.

By processing the FFT on $y_k$, we obtain:

$$Y_k = H_k \cdot X^c_k + n_k, \quad 0 \leq k \leq N - 1$$

$$= H_k \cdot X_k + Q_k$$

(2.12)

(2.13)

where $Q_k$ is the sum of the AWGN and the clipping noise and $H_k$ represents the complex channel gain on the $k$th subcarrier. Equalization is performed in the frequency domain using zero-forcing (ZF):

$$Z_k = \alpha_k Y_k, \quad \alpha_k = \frac{H^*_k}{|H_k|^2}$$

(2.14)

**Turbo-DAR**

These equalized symbols are used as the inputs of the Turbo-DAR algorithm. This algorithm, whose principle is given in Figure 2-4, implements an iterative technique summarized as follow:
1. First, the equalized signal $Z$ is stored in memory $\tilde{X}^{(0)} = Z$.

2. The noisy symbols $\tilde{X}^{(i)}$ are coded using the hard symbol estimator depicted in Figure 2-5:

$$P(b_k = a) = \sum_{s \in S} P(b_k = a | s_m) P(S = s_m) = \sum_{s_m \in S'} P(S = s_m)$$

where $S$ denotes the set of symbols of the M-QAM constellation and $S'$ the subset of $S$ such that $s_m = \{b_0, \ldots, b_{\log_2 M - 1}\}$ and $b_k = a$. These bit likelihoods are deinterleaved to provide the likelihoods of the noisy codeword bits $P(C_k(i))$ and are used at the Viterbi decoder input to compute the maximum likelihood decoded sequence $\hat{C}^{(i)}$ and an estimation of the information bits.

3. $\hat{C}$ is interleaved, mapped onto symbols and converted back to the time domain using an IFFT leading to $\tilde{X}^{(i)}$.

4. The detection of the clipped samples is performed in the time domain by comparing $|\tilde{X}^{(i)}|\leq A$.
5. The estimated symbols are converted to the frequency domain by an FFT. Go back to step 2 and increment the index number $i + 1$. Iterate $I$ times.

The final decision is taken using $\tilde{X}^{(l)}$ to obtain the estimated information bits by Viterbi decoding, Figure 2-5. A soft decision Turbo-DAR can be defined the same way, replacing the Viterbi decoder by a Bahl-Cocke-Jelinek-Raviv (BCJR) decoder with soft outputs.

Figure 2-6 illustrates the performance of the proposed techniques for $\text{CRa} = 1 \text{ dB}$ using QAM-16 on a Time Invariant Frequency Selective (TIFS) channel. Those results show a high improvement of the Bit Error Rate (BER) by combining the effect of the error correcting code and a decision taken in the frequency domain in a turbo fashion.

![Figure 2-6. BER comparison on TIFS channel of hard and soft Turbo-DAR with 16-QAM, CRa = 1 dB,](image.png)
Although current results are given for static channels, Turbo-DAR methods can also be used on time-varying channels such as in high rate wireless applications with adaptive channel estimator, e.g. [CiBi94], [MoMe01].

**Exceeding power: an alternative to PAPR**

It seems it is not entirely clear how well the PAPR measure is related to the real effects of nonlinearities [BeEr02]. Does a reduction of the PAPR always lead to a decrease in the effects of the non-linearity? In some recent contributions [Brai00], [BSGS02], other measures have also been mentioned for the sensitivity of multicarrier systems to non-linearity.

Classically, the PAPR measure is defined by:

\[
PAPR = \frac{\max |x_n|^2}{E[|x_n|^2]} \tag{2.17}\]

Theoretically, the maximum value of a sample \(x_n\) from an OFDM symbol can get as high as \(N\), but the probability of such a peak is very small, especially for a high number of subcarriers. In practice, in order to decrease the intercarrier interference and out-of-band radiation, it is required that the amplifier operates in its linear region. Therefore it is desirable to limit the maximum envelope of the multicarrier signal, namely the PAPR of the signal. On the other hand, reducing the PAPR does not necessarily mean an improvement in the performance of the system. However, there are other possibilities in defining a measure. A measure of the signal degradation in non-linearity should have a set of desirable properties:

- it should be independent on the non-linearity
- it should be easy to compute
- it should be highly related to the effects of a non-linearity.

The first property is desirable, since the non-linearity and the operating point of the system are typically unknown at the time of baseband system design; the second one is important in the sense that the measure must be easily attainable when using a limited set of data; the third one is important for obvious reasons. PAPR measure fulfils the first requirement; considering the PAPR of a discrete signal, it fulfils the second property as well; however, according to [TaJa00], the performance of an OFDM system with non-linearity depends on the power of distortion, while the peak value does not show the power of the signal or the distortion. This may discard PAPR measure for some applications. Here, Excess Power of the OFDM symbol is defined as the amount of the power over a certain power level:

\[
P_{Excess} = E[(|s_n| - |\tilde{s}_n|)^2(|s_n| > |\tilde{s}_n|)] \tag{2.18}\]

where \(|\tilde{s}_n|\) is the average envelope of the signal. The above definition is one example of a large set of measures that can be used for amplitude variations. A more general form of (2-18) can be written as:
where $L$ is the reference level for the distortion. It is straightforward to show that $P_{Excess}^{(m,L)}$ gives a peak of signal when $m = \infty$. Hence,

$$P_{APR} \propto \lim_{m \to \infty} P_{Excess}^{(m,L)}, \quad \forall L < \max |s_n|$$

(2.20)

On the other hand, $P_{Excess}^{(m,L)}$ can be a measure of power when $m = 2$ as defined in (2.7). Both PAPR and $P_{Excess}^{(2)}$ fulfill the two first properties above. Figure 2-7 pictures the correlation coefficients between the distortion power and PAPR, and distortion power and excess power are plotted for different values of back-off. This figure shows that when the non-linearity is severe, excess power can show the distortion very well, while for high back-offs PAPR is a better measure.

![Figure 2-7. Correlation coefficients between PAPR and $P_d$ (solid) and $P_{Excess}$ and $P_d$ (dashed) vs. Input Back-Off.](image)

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