Piezoelectric Technology: A Primer

James R. Phillips - August 10, 2000

Piezoelectric ceramics are used in myriad applications ranging from generating exquisitely controlled forces at the microscopic level to developing tremendous forces at the macro level. Piezoelectric ceramics, and in particular PZT ceramic, are used in inexpensive but rugged sensors as well as multi-layer transformers for generating high voltages without any EMI. In fact, there are too many applications to cover in a single article, but the basic principals in this primer can be used to both understand and design piezoelectric structures and devices.

Piezoelectric ceramics have unusual properties in both the electrical and mechanical domains. So if it's been a while since you shook hands with the dielectric constant or Hooke's Law, see Reviewing the Equations for High-Permittivity Dielectrics and Bridging the Mechanical and Electrical Worlds.

Piezoelectricity
The piezoelectric effect is a property that exists in many materials. The name combines "piezo," which is derived from the Greek word for pressure, and "electric" from the Greek word for amber—static electricity generated by rubbing amber being the first known electric phenomenon. A rough translation is pressure-electric effect. In a piezoelectric material, the application of a force or stress results in the development of a charge in the material. Conversely, the application of a charge to the same material will result in a change in mechanical dimensions or strain.

Several ceramic materials exhibit a piezoelectric effect. These include lead-zirconate-titanate (PZT), lead-titanate (PbTiO$_3$), lead-zirconate (PbZrO$_3$), and barium-titanate (BaTiO$_3$). Strictly speaking, these ceramics are not actually piezoelectric but rather exhibit a polarized electrostrictive effect.

A material must be formed as a single crystal to be truly piezoelectric. Ceramics have a multi-crystalline structure made up of large numbers of randomly orientated crystal grains. The random orientation of the grains results in a net cancelation of the piezoelectric effect. The ceramic must be polarized to align a majority of the individual grains' effects. Nonetheless, the term "piezoelectric" has become interchangeable with "polarized electrostrictive effect" in most literature.

Electrical/Mechanical Properties Change with Load
Piezoelectric materials exhibit a unique property—their dielectric constant varies with mechanical load and their Young's modulus varies with electrical load.
Dielectric Constant

\[ \varepsilon_r^{\text{FREE}} (1 - k^2) = \varepsilon_r^{\text{CLAMPED}} \]

The dielectric constant of a material reduces with mechanical load. "Free" stands for a state when the material is able to change dimensions with applied field. "Clamped" refers to either a condition where the material is physically clamped or is driven at a frequency high enough above mechanical resonance that the device can't respond to the changing E field.

Elastic Modulus (Young's Modulus)

\[ Y^{\text{OPEN}} (1 - k^2) = Y^{\text{SHORT}} \]

The elastic modulus means that the mechanical stiffness of the material reduces when the output is electrically shorted. This is important in that both the mechanical QM and resonant frequency will change with load. Young's Modulus is also the property that is used in the variable dampening applications, one of the many interesting applications for these materials.

Piezoelectric Equation

When a voltage is applied across a capacitor made of normal dielectric material, a charge results on the plates or electrodes of the capacitor. Charge can also be produced on the electrodes of a capacitor made of a piezoelectric material by the application of stress. This is known as the Direct Piezoelectric Effect. Conversely, the application of a field to the material will result in strain. This is known as the Inverse Piezoelectric Effect. The equation that defines this relationship is the piezoelectric equation:

\[ D_i = d_{ij} \sigma_j \quad \text{where:} \quad D_i = \text{Electric Displacement} \]
\[ (\text{or Charge Density}) \]
\[ d_{ij} = \text{Piezoelectric Modulus,} \]
\[ \text{the ratio of strain to applied field or charge density to applied mechanical stress} \]

Stated differently, d measures charge caused by a given force or deflection caused by a given voltage. We can also use this to define the piezoelectric equation in terms of field and strain:

\[ D_i = \frac{\sigma_i \chi_i}{E_j} \]

Electric displacement is defined as:

\[ D_i = \varepsilon_{ij} E_j \]
therefore,

\[ e_{ij} E_j = d_{ij} \sigma \]

and

\[ E_j = \frac{d_{ij} \sigma_j}{e_{ij}} \]

which results in a new constant:

\[ g_{ij} = \frac{d_{ij}}{e_{ij}} \]

This constant is known as the piezoelectric constant and is equal to the open circuit field developed per unit of applied stress or as the strain developed per unit of applied charge density or electric displacement. The constant can then be written as:

\[ g = \frac{\text{field}}{\text{stress}} = \frac{\text{volts / meter}}{\text{newtons / meter}^2} = \frac{\Delta L / L}{\varepsilon V / t} \]

Fortunately, many of the constants in the formulas above are equal to zero for PZT piezoelectric ceramics. The non-zero constants are:

\[ s_{11} = s_{22}, \; s_{33}, \; s_{12}, \; s_{13} = s_{23}, \; s_{44}, \; s_{66} = 2 (s_{11} - s_{12}) \]
\[ d_{31} = d_{32}, \; d_{33}, \; d_{15} = d_{24} \]

Basic Piezoelectric Modes
Piezoelectric ceramic materials are not piezoelectric until the random ferroelectric domains are aligned. This alignment is accomplished through a process known as poling. Poling consists of inducing a DC voltage across the material. The ferroelectric domains align to the induced field resulting in a net piezoelectric effect. It should be noted that not all the domains become exactly aligned. Some of the domains only partially align and some do not align at all. The number of domains that align depends upon the poling voltage, temperature, and the time the voltage is held on the material.

During poling the material permanently increases in dimension between the poling electrodes and decreases in dimensions parallel to the electrodes. The material can be de-poled by reversing the poling voltage, increasing the temperature beyond the material's Currie point, or by inducing a large mechanical stress.

Post Poling

Applied voltage:

Voltage applied to the electrodes at the same polarity as the original poling voltage results in a further increase in dimension between the electrodes and decreases the dimensions parallel to the electrodes. Applying a voltage to the electrodes in an opposite direction decreases the dimension between the electrodes and increases the dimensions parallel to the electrodes.

Applied force:

Applying a compressive force in the direction of poling (perpendicular to the poling electrodes) or a
tensile force parallel to the poling direction results in a voltage generated on the electrodes that has the same polarity as the original poling voltage. A tensile force applied perpendicular to the electrodes or a compressive force applied parallel to the electrodes results in a voltage of opposite polarity.

Shear:

Removing the poling electrodes and applying a field perpendicular to the poling direction on a new set of electrodes will result in mechanical shear. Physically shearing the ceramic will produce a voltage on the new electrodes.

**Piezoelectric Benders**

Piezoelectric benders are often used to create actuators with large displacement capabilities. The bender works similarly to the action of a bimetallic spring. Two separate bars or wafers of piezoelectric material are metallized and poled in the thickness expansion mode. They are then assembled in a +–+– stack and mechanically bonded. In some cases, a thin membrane is placed between the two wafers. The outer electrodes are connected together and a field is applied between the inner and outer electrodes.

As a result, for one wafer the field is in the same direction as the poling voltage while the other is opposite to the poling direction. Therefore, one wafer is increasing in thickness and decreasing in length while the other wafer is decreasing in thickness and increasing in length. This results in a bending moment.

Loss

There are two sources for loss in a piezoelectric device: mechanical and electrical.

\[
\text{Mechanical loss: } Q_M = \frac{\text{Mechanical stiffness reactance or mass reactance}}{\text{Mechanical resistance}}
\]

\[
\text{Electrical loss: } \tan \delta = \frac{\text{Effective series resistance}}{\text{Effective series reactance}}
\]

Simplified Piezoelectric Element Equivalent Circuit
\[ R_i = \text{Electrical resistance} \]
\[ C_i = \text{Input capacitance} = \frac{\varepsilon_0 \varepsilon_r A}{t} \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ farads / meter} \]
\[ A = \text{Electrode Area} \]
\[ t = \text{Dielectric Thickness} \]
\[ L_M = \text{Mass (kg)} \]
\[ C_M = \text{Mechanical compliance} = \frac{1}{\text{Spring rate (M / N)}} \]
\[ N = \text{Electro-mechanical linear transducer ratio (newtons/volt or coulomb/meter)} \]

This model has been simplified and it is missing several factors. It is only valid up to and slightly beyond resonance. The first major problem with the model is related to the mechanical compliance \( C_M \). Compliance is a function of mounting, shape, deformation mode (thickness, free bend, cantilever, etc.), and modulus of elasticity. The modulus of elasticity is, however, anisotropic and it varies with electrical load. The second issue is that the resistance due to mechanical \( Q_M \) has been left out. Finally, there are many resonant modes in the elements, each of which has its own \( C_M \) as shown below.

**Mechanical Compliance**

Mechanical compliance, which is the inverse of spring constant, is a function of the shape, mounting method, modulus, and type of load. Some simple examples are shown below.

**Simple Beam—Uniform end load**
Simple Beam—Uniform load—End mounts

$$C_M = \frac{W}{A} = \frac{A}{W \cdot t}$$

$$I_M = \rho \cdot A \cdot W \cdot t$$

Simple Beam—Uniform load—Cantilever mount

$$C_M = \frac{5}{324} \frac{A^3}{Y_d \cdot I}$$

$$I = \text{Moment of inertia} = \frac{W \cdot r^3}{12}$$

$$C_M = \frac{5A^3}{32 \cdot W \cdot r^3}$$

The various elements that have been explained can now be combined into the design of a complete piezoelectric device. The simple piezoelectric stack transformer will be used to demonstrate the way they are combined to create a functional model.

Piezoelectric Transformers
The piezoelectric transformer is constructed of multilayer PZT ceramic. The manufacture of the transformer is similar to the manufacture of ceramic chip capacitors. Layers of flexible, unfired PZT ceramic tape are printed with metallic patterns and then aligned and stacked to form the required structure. The stacks are then pressed, diced, and fired to create the final ceramic device.

Simple Stack Piezoelectric Transformer
The piezoelectric transformer acts as an ideal tool to explain the modeling of piezoelectric devices in
that it utilizes both the direct and indirect piezoelectric effects. The transformer operates by first converting electrical energy into mechanical energy in one half of the transformer. This energy is in the form of a vibration at the acoustic resonance of the device. The mechanical energy produced is then mechanically coupled into the second half of the transformer. The second half of the transformer then reconverts the mechanical energy into electrical energy. The figure below shows the basic layout of a stack transformer. The transformer is driven across the lower half (dimension $d_1$) resulting in a thickness mode vibration. This vibration is coupled into the upper half and the output voltage is taken across the thinner dimension ($d_2$).

The equivalent circuit model for the transformer (shown above) can be thought of as two piezoelectric elements that are assembled back to back. These devices are connected together by an ideal transformer representing the mechanical coupling between the upper and lower halves. The input resistance ($R_i$) and the output resistance ($R_o$) are generally very large and have been left out in this model. The resistor ($R_L$) represents the applied load. Determining the values of the various components can be calculated as shown previously.

**Input / Output capacitance:**

\[
C_i = \varepsilon_0 \varepsilon_r \frac{\text{Input area}}{\text{Input thickness}} = \varepsilon_0 \varepsilon_r \frac{A}{d_1} \frac{W}{d_1}
\]

similarly,

\[
C_o = \varepsilon_0 \varepsilon_r \frac{\text{Output area}}{\text{Output thickness}} = \varepsilon_0 \varepsilon_r \frac{nA}{d_2} \frac{W}{d_2}
\]

**Mechanical compliance:**

The mechanical compliance ($C_m$) can be represented by a simple beam subjected to a uniform axial load. This is because the thickness expansion mode will apply uniform stress across the surface. It should be noted that the beam length is measured with respect to the vibration node. The vibration node is used as this is the surface that does not move at resonance and can, therefore, be thought of...
as a fixed mounting surface.

\[ C_M = \frac{\text{Beam length}}{\text{Beam area}} \gamma_{33} \]

\[ C_{M1} = \frac{d_1}{AWY_{33}} \]

\[ C_{M2} = \frac{d_2}{AWY_{33}} \]

**Note:** Even if \( nd_2 \neq d_1 \) the vibration node will still be located in the mechanical center of the transformer.

**Mass:**

\[ L_{M1} = \rho A W d_1 \]

\[ L_{M2} = \rho A W nd_2 = \rho A W d_1 \]

**Resistance:**

The resistances in the model are a function of the mechanical QM and Q of the material at resonance and will be calculated later.

**Ideal transformer ratio:**

The transformer ratio \( (N_1) \) can be thought of as the ratio of electrical energy input to the resulting mechanical energy output. This term will then take the form of newtons per volt and can be derived from the piezoelectric constant \( (g) \).

as before:

\[ g = \frac{\text{Electric Field}}{\text{Stress}} = \frac{\text{volts/meter}}{\text{newtons/meter}^2} \]

therefore:

\[ \frac{1}{g} = \frac{N}{m} \]

\[ \frac{1}{g} = \frac{N}{m} \]

\[ N_1 = \frac{1}{g} \frac{\text{Area of applied force}}{\text{Length of generated field}} \]
The output section converts mechanical energy back to electrical energy and the ratio would normally be calculated in an inverse fashion to $N_1$. In the model, however, the transformer ratio is shown as $N_2 : 1$. This results in a calculation for $N_2$ that is identical to the calculation of $N_1$.

$$N_2 = \frac{\text{Area of applied force}}{\text{Length of generated field}}$$

or

$$N_2 = \frac{A \cdot W}{g_{33} \cdot d_1}$$

The transformer $1 : N_C$, represents the mechanical coupling between the two halves of the transformer. The stack transformer is tightly coupled and the directions of stress are the same in both halves. This results in $N_C \approx 1$.

Model Simplification:

The response of the transformer can be calculated from this model, but it is possible to simplify the model through a series of simple network conversions and end up in an equivalent circuit whose form is the same as that of a standard magnetic transformer.

where, due to translation through the transformer,

$$C_{M2} = N_C^2 C_{M2} \text{ and } L_{M2} = L_{M2} / N_C^2$$

but $N_C^2 \approx 1$, therefore

$$C_{M2} = C_{M2} \text{ and } L_{M2} = L_{M2} = L_{M1}$$
which allows the next level of simplification

\[ L' = L_{M1} + L_{M2}' = 2L_1 = 2 \rho A W d_i \]

\[ C' = \frac{C_{M1}C_{M2}}{C_{M1} + C_{M2}'} = \frac{CM1^2}{2CM1} = \frac{CM1}{2} = \frac{d_1}{2A/W/Y_{33}} \]

**Final simplification**

where

\[ C = C' N_1^2 \text{ and } L = L' / N_1^2 \]

and, from before

\[ N_1 = \frac{A/W}{q_{33}d_1} \]

therefore
The last value we need to calculate is the motional resistance. This value is based upon the mechanical QM of the material and the acoustic resonant frequency.

Resonant frequency

\[
\omega_0 = \frac{1}{\sqrt{L/C}} = \frac{1}{\sqrt{\frac{2 \rho d_1 g_{33}^2 d_1^2 A W}{A W} \frac{A W}{2 Y_{33} g_{33}^2 d_1}}} = \frac{1}{\sqrt{\frac{\rho d_1^2}{Y_{33}}}} = \frac{1}{d_1 \sqrt{\frac{\rho}{Y_{33}}}}
\]

\[c_{PZT} = \text{speed of sound in PZT} = \sqrt{\frac{Y}{\rho}}\]

therefore

\[
\omega_0 = \frac{c_{PZT}}{d_1}
\]

The equation shown above states that the resonant frequency is equal to the speed of sound in the material divided by the acoustic length of the device. This is the definition of acoustic resonance and acts as a good check for the model. The final derivation is the value of resistance:

\[Q_M = 1 / \omega_0 R C\]

or
\[ R = \frac{1}{\omega_0 Q_M C} \]

\[
R = \frac{d_1 \sqrt{\rho / Y_{33}}}{Q_M} \frac{2 Y_{33} g_{33}^2 d_1}{\Lambda W} = \frac{2 d_1^2 g_{33}^2 \sqrt{\rho Y_{33}}}{Q_M \Lambda W}
\]

**Note:** \( C_M \) and \( R \) are both functions of \( Y_{33} \) and \( Y_{33} \) is a function of \( R_L \)

It should be noted that the model is only valid for transformers driven at or near their fundamental resonant frequencies. This is because the initial mechanical model assumed a single vibration node located at the center of the stack, which is only true when the transformer is driven at fundamental resonance. There are more nodes when the transformer is driven at harmonic frequencies.

**Note:** Stress is 90° out of phase from displacement

There are no fixed nodes at frequencies other than resonance. This means that the transformer must be designed with the resonate mode in mind or phase cancellations will occur and there will be little or no voltage gain. It is often difficult to understand the concept of nodes and phase cancellation, so a simple analogy can be used. In this case, waves created in a waterbed will be used to explain the effect.

Pressing on the end of a waterbed creates a wave of displacement that travels down the length of the bed until it reaches the opposite end and bounces back. The water pressure (stress) is the lowest, or negative with respect to the water at rest, at a point just in front of the wave and highest at a point just behind the wave. The pressures at the crest and in the trough are at the same pressure as the bed at rest.

The wave will reflect back and forth until resistance to flow causes it to damp out. The average pressure over time at any point in the bed will be exactly the same as the pressure at rest. Similarly, the average stress in a transformer off resonance will approach zero and there will be no net output.
Pressing on the end of the same bed repeatedly just after the wave has traveled down the length, reflected off the end, returned, and reflected off the driven end will result in a standing wave. This means that one half of the bed is getting thicker as the other half is getting thinner and the center of the bed will be stationary. The center is the node and the thickness plotted over time of either end will form a sine wave. There will be no net pressure difference in the center, but the ends will have a pressure wave that forms a sine wave 90° out of phase with the displacement. The piezoelectric transformer again works in the same manner with no voltage at the node and an AC voltage at the ends. It is fairly simple to expand this concept to harmonics and to other resonate shapes.