Combining standard component values improves circuit designs

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A previous EDN article proposed a technique for modifying active-RC-filter-design equations to use exact standard-component values (Reference 1). The main idea was to determine standard-component-value ratios that you could implement for your desired transfer-function coefficients. The article presented a matched-filter design with selectable rates and showed that the technique worked with different component factors (component scaling of 10, 100, or 1000) and tuning elements (the capacitors) as functions of rate. Although this approach is viable for many active-filter implementations, standard component values are not always enough. Indeed, the design of board-level electronic circuits typically results in ideal passive-component values to achieve a desired response. Using standard component values often distorts the circuit’s response from the ideal and can fail to meet the design’s requirements. You can employ tuning elements in critical circuit stages, but this postdesign adjustment can be time-consuming for complex modules or high-frequency applications. Even when the response distortion is initially acceptable, component variation and drift can subsequently affect the circuit’s accuracy.

Statistical methods can significantly alleviate the challenges that component variations present. Techniques such as ANOVA (analysis of variance) (Reference 2), matrix experiments, designed experiments (Reference 3), and orthogonal arrays (Reference 4) provide tools to bring designs under statistical control. The complementary problem of response distortion has been a topic of interest in filter design. Predistortion, which designers in the late 1930s first applied to insertion-loss filters, accounts for the circuit elements' parasitic parameters before design (Reference 5). More recent efforts have concentrated on active networks with nonideal amplifiers (Reference 6).

Other techniques, such as sensitivity and tolerance analyses, can provide information on a design’s critical elements when high-order effects are not critical (Reference 7). By incorporating sensitivity analysis and search techniques, tuning algorithms can assist you in meeting design objectives (Reference 8). Tuning can also incorporate a genetic-algorithm strategy (Reference 9).

These approaches, whose foundation is fairly complex network theory, present a significant computational load. This approach uses combinations of standard component values to approximate the desired (and critical) ideal values within predefined error limits. This approach uses only discrete resistors, inductors, and capacitors; it does not use tuning elements, such as potentiometers or voltage-variable capacitors. The supporting mathematics concentrate mainly on a simplified statistical analysis of the resulting component combinations. A Chebyshev-filter-design example illustrates the techniques.
Component considerations

Suppose that you seek to approximate an ideal passive-component value by some numeric combination of standard component values. Ideally, the result is a replacement of the ideal component value by two or more passive standard values approximating the ideal value. To avoid arbitrarily increasing the parts count, the method uses only two standard-value components in place of each ideal-value component. This approach both limits the parts count and simplifies the subsequent analysis.

The passive elements, that is, the resistors, capacitors, and inductors, have specific characteristics that require attention when you combine them to meet design requirements (Reference 10). You can analyze the features as they relate to two possible methods of combining two elements: in series; selecting impedances as \( z = z_1 + z_2 \); and in parallel, substituting admittances as \( 1/z = 1/z_1 + 1/z_2 \).

Resistors can have precise values and tight tolerances. Thus, you can combine resistors in series or in parallel. The only immediate consideration is power rating. Combining two resistors in parallel to achieve a desired value requires that each device has greater resistance than the original; the current is correspondingly reduced in each, resulting in lower power dissipation for each resistor. However, you must remember that higher resistance contributes noise to a circuit.

In filtering applications, the most important characteristics of an inductor are \( Q \) and SRF (series resonant frequency). Smaller inductor values typically have SRF greater than 1 GHz and higher than 80 Q versus frequency. Smaller inductor values also exhibit nearly constant inductance, \( L \), versus frequency and can tolerate higher dc current (Reference 11). Therefore, the preferred combinations for inductors are smaller values in series. The only immediate exception is combining larger values to approximate small inductors with high tolerances. In each case, you must exercise great care in placing inductors on pc boards; coupling between inductors can significantly affect the behavior of the combined components. Normally, inductors should be the last components that you consider for component-value combinations.

You can choose among many capacitor types for different applications. Again, because this analysis emphasizes filtering, it concentrates on ceramic capacitors, particularly NP0 types. These capacitors have typical insulation resistance of \( 5 \times 10^3 \) MΩ-µF at 25°C and dissipation factor of 0.02% (with corresponding \( Q = 1/dissipation \) factor) (Reference 7). The SRF increases for lower capacitance values. Low-value capacitors, like low-value inductors, have higher tolerance. In addition, parallel combination places the same voltage across each device, so no voltage division exists to improve reliability. For approximating small values, you should consider placing capacitors in series.

In summary, the preferred combinations for each component are:

- **Resistors**: Combine standard values that result in the closest approximation to the ideal value.
- **Inductors**: For most values, combine inductors in series. For very small inductances, combine inductors in parallel.
- **Capacitors**: For most values, combine capacitors in parallel. For very small values (in which tolerance increases significantly), combine capacitors in series.

Statistical analysis

As mentioned, you can combine two passive components either as a series summation of impedances or as a parallel summation of admittances. Therefore, you should analyze the summation of two random variables, \( x \) and \( y \), as \( z = x + y \). Reference 12 treats this approach, which the following
The pdf (probability density function) of \( z \) is given by

\[
f_z(z) \, dz = \int_{-\infty}^{\infty} f_z(z-y) \, dy \, dz. \tag{1}\]

If you assume the random variables \( x \) and \( y \) are independent, then you can simplify the joint probability of Equation 1 by \( f(x,y)=f_x(x) \) and \( f_y(y) \), and Equation 1 becomes

\[
f_z(z) = \int_{-\infty}^{\infty} f_z(z-y) \, f_y(y) \, dy. \tag{2}\]

This integral is the convolution of the pdf's \( f_x(x) \) and \( f_y(y) \). Therefore, the density of the sum of two independent random variables equals the convolution of their densities.

Now, consider that impedances \( z_1 \) and \( z_2 \) are independent and uniformly distributed, with \( z_1 \) in \((a, b)\) and \( z_2 \) in \((c, d)\). If each has the same tolerance, it is trivial to show that the density of their sum \( z = z_1 + z_2 \) is a triangle (Figure 1). Although the net spread ranges from \( a+c \) to \( b+d \), the convolution centers the pdf with the peak exactly halfway between the extreme variables. If the pdf of the components were, instead, Gaussian, the convolution would again produce a pdf with peaking at the center of the spread resulting from the convolution.

For summing admittances \( y = y_1 + y_2 \), you still have a convolution of the pdfs as in the case for impedances. The only difference is that the pdfs are defined in terms of impedance spread, so the pdfs of the admittance are a result of the inversion of the impedance. For any single admittance, \( y \), the density now becomes

\[
f_y(y) = \frac{1}{y^2} f_x\left(\frac{1}{y}\right). \tag{3}\]

Therefore, the density scales by the factor \( 1/y^2 \) with the resulting convolution as shown in Figure 2. Again, the new pdf has peaking at the center of the distribution.

Note that the worst-case combination of elements does not change the circuit's statistical bounds. That is, as tolerance analysis demonstrates, if all of the components reach their extreme values, combining components does not reduce the statistical spread. However, the extreme cases are rare, and the Monte Carlo simulations that follow clearly show that, for the same tolerances, the combination of components offers better statistical behavior than the single component.

**Combining component values**

Engineers have used alternative configurations to combine standard component values and to achieve a desired response. However, no formal approach easily lends itself to determining the closeness of the answer or to figuring other acceptable combinations.

The series combination of components offers some interesting opportunities for approximating an ideal value. You should first consider straight combinations of standard values. Table 1 provides the sums of standard 5%-tolerance values. Each entry in the table is the direct sum of the values in bold along the first row and first column.

From Table 1, you can first conclude that because you are adding two values within the same range, at least two identical values exist. For example, \( 91 + 10 = 10 + 91 = 101 \). The entries in Table 1 are
symmetrical about the diagonal. Also, certain values appear many times in the table. For example, 86, the most frequent entry, appears 11 times, and 10 entries have a value of 69. Nine entries have values 40 and 78, and eight have values 46, 51 (good for RF), 57, 80, and 92.

Another means of combining values in series is to use one standard value and add one-tenth scale of some other value. Table 2 takes this approach. In this case, the entry in the table is the sum of the bold value in the row and one-tenth of the bold value in the column. Most entries are dissimilar. However, Table 2 provides several opportunities for approximating ideal values that you should exploit when the entries in Table 1 aren’t sufficiently close to what you need.

To use these tables to determine the optimal series combination for some value, assume that you need to find 45.7Ω. The following are your options:

- The nearest standard value is 47, which gives 2.8% relative error.
- From Table 1, you can use any of eight combinations of 46Ω, which reduces the relative error to 0.6%.
- From Table 2, you can use 43+2.7, which is an exact fit. If these standard values are unavailable, you can use 39+6.8=45.8, yielding a relative error of only 0.2%.

You can use tables 1 and 2 to find series combinations of only selected values if not all values are available. Tables 1 and 2 are likely to prove most useful for this purpose.

Parallel combinations

Now consider the parallel combination of components. The parallel combination of two impedances is

$$z = \frac{z_1 z_2}{z_1 + z_2} = \frac{1}{1 + z_1 / z_2}. \tag{4}$$

Equation 4 scales one of the values by the ratio of the standard component values used. The key factor is the ratio $z_1/z_2$. Table 3 provides this ratio for standard values of 5% tolerance. Table 4 provides the factor $1/(z_1 + z_2)$ for the same standard values.

Use these tables in the following way: For ratios used in calculations of parameters, such as Q, use the straight ratios of Table 1. To use the factors in Table 2, find the desired value in the table and use the bold values in the column and row that correspond to that entry. For example, if the desired value is 10.44Ω, pick the entry 10.43 in Table 4, which yields a 0.1% relative error. This entry corresponds to the parallel combination of 30 and 16.

Wherever you combine component values, you add an element to the circuit. Therefore, you should use the technique with care and consider it only for the components that contribute the most to variations in the circuit response. Good candidates also include small capacitor and inductor values with large tolerances. For active filters that consist of cascaded biquadratic stages, the highest-Q stages should be prime candidates.

Chebyshev-filter-design example

Consider the following requirement for an all-pole equiripple Chebyshev filter: Passband-ripple magnitude $M_P=1$ dB at $f_P=500$ kHz. Stopband-rejection magnitude $M_S=40$ dB at $f_S=1.1$ MHz. To determine the filter order, use the ripple parameter, $\varepsilon < \sqrt{\frac{M_P}{10^\alpha - 1}}$ for $x=p$ (passband), $s$ (stopband),
along with the stopband corner ratio \( \Omega_s = f_s / f_p \) (Reference 7).

\[
N = \frac{\cosh^{-1}(\frac{s_p}{s_p^*})}{\cosh^{-1}(\frac{f_s}{f_p})}.
\]

The resulting filter order is \( n = 5 \). Figure 3 shows the filter's passband response with ideal component values.

The stopband corner ratio of \( \Omega_s = f_s / f_p = 2.2 \) provides excess rejection for the given order. You can trade this rejection to reduce the passband ripple without modifying any other parameters or increasing the filter order. Using the filter nomograph and technique, you can minimize the passband ripple to \( M_p = 0.1 \text{ dB} \) and satisfy all other requirements while holding the filter order fixed (Reference 13). However, the approach reduces the filter's selectivity and increases the sensitivity of the selectivity to the passband-ripple parameter (Reference 14).

\[
\frac{\Omega_s}{\Omega_p} = 2 - \frac{3}{1 + s_p^2}
\]

Therefore, you can use one of the following methods to improve the design: LC and active-RC implementations. The LC standard values for a Chebyshev filter with 0.1-dB ripple and order \( n = 5 \) are \( Q = 1.1468 \), \( L_2 = 1.3712 \), \( C_3 = 1.9750 \), \( L_4 = 1.3712 \), and \( C_5 = 1.1468 \) (Reference 15). You obtain these values when \( R_L = R_S \), where \( R_L \) is the load resistance and \( R_S \) is the source resistance. Performing the frequency and impedance scaling for the desired characteristics yields the ladder filter of Figure 4. The values in parentheses are the nearest standard values for each part. Figure 5 shows the Chebyshev filter's simulated passband response with standard values (Reference 16).

Assuming a 5% Gaussian deviation for all of the components, Figure 6 shows the corresponding response as computed in 20 Monte Carlo runs. The peak passband-ripple variation increases to greater than 0.2 dB. However, you can replace the 7.5-nF capacitors with two parallel capacitors of 5.2 and 2.1 nF (Figure 7). The resulting value, 7.3 nF, is exactly what you need. Figure 8 shows the new batch of Monte Carlo runs. The peak ripple variation is less than 0.15 dB, and the actual variation is more centered.

To implement this design using an active RC filter, you must first commit to the design to identify opportunities for improvement. Figure 9 shows one such design with ideal and nearest standard values. The design consists of a one-pole lowpass stage followed by cascaded moderate-sensitivity lowpass designs (Reference 17). In this case, the design uses standard values for all of the capacitors, so you need to modify only the resistors. Nevertheless, the first stage has a value that is midway between the nearest standard values. Therefore, you must modify this resistor as well as other parts of the circuit.

**Eliminating distorted response**

Figure 10 shows the circuit's response with the nearest 5%-tolerance standard values. The passband exhibits considerable distortion, especially because of the selection of the resistor for the first pole, which somewhat narrows the response. Assuming a 5% Gaussian distribution of all of the components, Figure 11 shows the corresponding 20 Monte Carlo runs. The response variation is significant, with a peak deviation greater than approximately 1.8 dB in addition to the original distortion.
When improving the circuit response, you may notice that the first stage pole is at too low a frequency. Therefore, you should make the resistance as close as possible to the ideal value. In this case, the resulting value is 590Ω—the series combination of 390 and 200Ω. Leave the second stage alone, because it has relatively low Q. The final stage with high Q is the next candidate. Modify all the resistor values of this stage, making \( R_5 = R_7 = 6100\Omega = (3900 + 2200) \) and \( R_6 = 1380\Omega = (820 + 560) \). The modified design includes four additional resistors (Figure 12).

Figure 13 shows simulation results of the modified active RC filter with 5% Gaussian distribution. The response variation in most of the stopband is now less than 1 dB, a 0.8-dB improvement. With the first stage pole moved to its proper location, the response shows more peaking near the passband edge. Peak deviation is still about 1.8 dB but is confined to smaller portions of the passband. You can further improve the circuit response by slightly reducing the frequency of the first-stage pole, but this reduction introduces distortion toward the lower end of the passband.

With the techniques described in this article, you can use standard-value components to construct passive and active circuits whose designs specify nearly ideal values. The resulting circuits not only exhibit lower response distortion but also possess superior immunity to component variations in most practical cases and provide higher yield with only a modest increase in component count.

References