Understand Stepper Parameters Before Making Measurements

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Stepper motors offer a bundle of tradeoffs that can give test engineers headaches when they need to determine why a particular motor fails in a specific application. Stepper motors excel at providing precise angular velocity and position control in response to voltage pulses from open-loop control circuitry, but only when the system designer has accounted for the effects of the relevant motor parameters.

In a typical stepper motor, a multitoothed iron rotor responds to varying magnetic fields established in stator electromagnets by control circuits. These circuits must provide the necessary stator phase-current pulses in the proper sequence to drive the rotor to a specified angular position in a specified time and with a specified accuracy, and to hold it there until control circuitry commands a new position. These motor parameters are relevant to whether or not a motor can meet the specified performance requirements:

- dynamic torque—the torque developed by the motor when in motion; dependent on the current in stator electromagnets;
- phase inductance—the electrical parameter that limits phase-current risetime and, hence, dynamic torque;
- holding torque—the torque a motor can develop to prevent a static load from pulling the system out of step;
- torque stiffness—the motor’s ability to resist angular displacements within a step; and
- rotor inertia—the mechanical parameter that limits motor acceleration and deceleration.

Stepper-motor problems often begin when a customer approaches a stepper-motor supplier with insufficient or inaccurate data. For example, a designer in your company might request a specific holding torque, when what’s really wanted is a specific dynamic torque. When the specified motor fails, the designer might specify a motor with higher holding torque.

That motor won’t necessarily work either, because a higher holding torque does not necessarily translate into a higher dynamic torque. In addition, a motor with higher holding torque always comes with a larger rotor inertia, which impedes acceleration and could slow performance to unacceptable levels. Despite the many variables, you can follow some quick guidelines to determine whether failed stepper motors simply reflect a bad batch of parts or are fundamentally unsuited to the application they’re serving.

Step accuracy depends on motor construction and is proportional to the motor’s torque stiffness. The relationship between holding torque, $T$, and angle displacement, $q$, is approximately

$$T = T_0 \cdot \sin(Nq)$$
where $N$ is the number of rotor teeth and $T_0$ is the maximum static holding torque. Torque stiffness is

then

$$\frac{dT}{dq} = N \cdot T_0 \cdot \cos(Nq)$$

where $dT$ is change in torque and $dq$ is change in angle. These equations show that increasing $N$ or $T_0$ are the only ways to improve torque stiffness. Creating additional electrical steps (microstepping) between primary mechanical steps (corresponding to rotor teeth) or increasing the number of phases (by employing 3-phase or 5-phase control, for instance) does not improve torque stiffness (Fig. 1).

![Diagram showing torque stiffness dependence on rotor teeth and maximum holding torque.](image)

**Figure 1.** Torque stiffness depends on a stepper motor’s number of rotor teeth and on its maximum holding torque.

Mechanical design parameters, manufacturing methods, and tolerances affect the absolute accuracy of a stepper motor. Step accuracy, for instance, depends on accurate stator teeth location, uniform rotor teeth distribution, and uniform air gaps. Manufacturers can achieve ±5% fundamental step accuracy: ±5% error on a 1.8° per step motor is 65.4 arc minutes.

This absolute error stays the same no matter how many microsteps a stepper-motor controller inserts. Microstepping increases step resolution, but not step accuracy. With microstepping, a ±5% absolute step error becomes a ±10% step error for half-stepping or a ±160% error for 32-stepping (Fig. 2). At step-error levels above ±100%, you can’t guarantee that the motor will step when it receives a pulse.

With today’s technology, microstepping beyond 32-stepping yields little if any improvements. If you are observing unsatisfactory stepper-motor accuracy at higher levels of microstepping, you’re probably observing the results of a flawed design—not a bad motor or controller.

**Current Risetime and Torque**
The major contributor to a stepper motor’s loss of running torque at high speeds is the fact that pulse switching
Figure 2. Microstepping will improve stepper-motor resolution but not accuracy. A ±5% absolute step error becomes a ±160% error at 32nd-step microstepping.

times can become shorter than motor phase-current risetimes; that is, the pulse ends before phase current rises to a level that provides sufficient torque. A simple RL series circuit’s mesh equations illustrate the relationships involved, where $t$ is current risetime:

\[
\begin{align*}
V &= IR + L \frac{dI}{dt} \\
I(t) &= \frac{V}{R} \left(1 - e^{-\frac{Rt}{L}}\right) \\
t &= \frac{L}{R} \ln \left(1 - \frac{R \cdot I(t)}{V}\right)
\end{align*}
\]

To reduce $t$, the power-supply voltage, $V$, must be as high as possible and the motor phase inductance, $L$, and resistance, $R$, must be as low as possible. **Figure 3a**, where $t = L/R$ is the time constant, shows current vs. time for the series RL circuit. Note that at time $t$ current has risen to $1-e^{-1}$ times its final level, $I_o$, or 0.63$I_o$, but only to $1-e^{-0.5}$ times $I_o$, or 0.39$I_o$, at time $t/2$. Because torque is approximately proportional to current, the torque available at speed $1/t_1$ (Fig. 3b), where $t_1 = t$, is 62% (or 100% x 0.63$I_o$/0.39$I_o$) greater than at the higher speed $1/t_2$ (Fig. 3c), where $t_2 = t/2$. Torque is optimized if phase current is applied for at least 2.3$t$, at which time the current has risen to 90% of $I_o$. 
Figure 3. (a) Motor phase inductance, L, and resistance, R, limit phase-current risetime. For example, application of phase voltage for t/2, t, and 2.3t, respectively, results in peak currents reaching 39%, 63%, and 90% of final value \(I_o\). Because torque is approximately proportional to current, motors that operate at (b) lower speeds, which permit longer applications of phase voltage, deliver more torque than those (c) operating at higher speeds.

Holding vs. Dynamic Torque
Assume that full current rise is obtained during phase switching. You can calculate the maximum dynamic torque of a 2-phase bipolar motor as follows:

\[
\overline{T_{\text{DYN}}} = T_o \int_{\pi/2}^{\pi} T(\theta) d\theta / (\pi/2)
\]

\[
= T_o \left[ \frac{3\omega/4}{\pi/2} \sin(\theta) d\theta + (\omega/2) d\cos(\theta) \right] / \pi/2
\]

\[
= T_o \left[ -\frac{\cos(\theta / 4) + \cos(\pi / 2) - \sin(\theta) + \sin(3\pi / 4)}{\pi / 2} \right]
\]

\[
= 0.90T_o
\]

where \(T_o\) is the maximum holding torque (Fig. 4a).

If you need to approach this ideal level, which requires perfect switching time relative to rotor position, you should employ closed-loop control. Open-loop control for the 2-phase bipolar motor will give you at least this minimum dynamic torque:

\[
\overline{T_{\text{MIN}}} = T_o \int_{\pi/2}^{\pi} T(\theta) d\theta / (\pi/2)
\]

\[
= T_o \int_{\pi/2}^{\pi} \sin(\theta) d\theta / (\pi/2)
\]

\[
= T_o \left[ -\cos(\theta) - \cos(\pi / 2) \right] / \pi/2 = 0.63T_o
\]

or 63.7% of \(T_o\) (Fig. 4b). Typical dynamic torque is \((90\% + 63.7\%)/2\) or 76.8%. 
Figure 4. The shaded portions of these curves represent (a) the maximum and (b) the minimum available dynamic torque, averaged between points a and b.

Given a certain power input, you can’t change a specific motor’s power output, \( \delta T(w) \, dw \) (where \( T(w) \) is the torque at angular velocity \( w \)). If your test instructions call for more dynamic torque than \( T(w) \) at angular velocity \( w \), then you are once again dealing with a flawed design, not bad components. But if you know your application’s speed, accuracy, resolution, and torque requirements, you can work with your firm’s designers to determine whether a specific stepper motor delivers sufficient torque at the desired operating speed within your specific mechanical constraints.

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