Ripples arise while characterizing devices at RF and microwave frequencies. RF engineers need to make sure that measurement set-up is properly calibrated and matched in order to avoid measurement errors due to ripples. Unmatched and improper interconnects, cables, connectors, SMA launches, and so on, in the set-up cause ripples that lead to measurement errors in device performance parameters, such as gain, output power, OIP3, return loss, and OIP2. Impedance mismatch within the cable, evaluation board traces, and package cause multiple reflections of electromagnetic fields, resulting in the formation of ripples. Thus, for RF device characterization, care is taken with proper measurement set-up to minimize such errors. Readers will gain a detailed understanding of theoretical analysis behind the formation of these ripples in this report. Also, lab measurements supported by some basic simulations are discussed.

Sometimes ripples are seen during RF device parameters characterization such as gain, linearity, and return loss. These ripples arise due to multiple reflections of the signal travelling within cables, connectors, evaluation board traces, device under test (DUT) and package. These ripples are caused due to impedance mismatch at the junctions of these interconnects.
Figure 1a shows a basic transmission line with source $V_S$, source impedance $Z_S$, transmission line characteristic impedance $Z_O$ and load impedance $Z_L$. In order for an input incident wave to travel completely, the transmission line should be matched to source and load, for example $Z_S = Z_O = Z_L = 50$ Ohms. If the transmission line, which could be a co-axial line as shown in Figure 1b or microstrip line as shown in Figure 2b, has characteristic impedance not equal to 50 Ohms, then there will be reflection at the planes of mismatch. This plane of mismatch could be considered as a boundary of two mediums with different dielectric properties. The portion of transmission line where characteristic impedance is not equal to 50 Ohms could be represented as medium with absolute permittivity $\varepsilon_2$, and the 50 Ohms source and load could be represented as medium with absolute permittivity $\varepsilon_1$ (Figures 1d and 1e).

Reflections due to impedance mismatch can better be understood by looking into electromagnetic wave interaction at the impedance mismatch planes. The interaction of the electromagnetic wave at these planes leads to the reflection and transmission of waves at the boundary of medium, which is quantized in terms of reflection coefficient $\Gamma$ and transmission coefficient $\tau$, respectively. Reflection coefficient is the ratio reflected $E_r$ and incident $E_i$ electric field strength. Transmission coefficient is the ratio of transmitted $E_t$ and incident $E_i$ electric field strength:

$$\Gamma = \frac{E_r}{E_i} \quad \text{(Eq. 1)}$$

$$\tau = \frac{E_t}{E_i} \quad \text{(Eq. 2)}$$

These coefficients are directly related to gain, output power, linearity, and return loss. In order to understand the ripples that occur due to impedance mismatch, it is necessary to understand reflection and transmission coefficients and the interaction of electromagnetic field at the boundaries of impedance mismatch. Any reflections in these coefficients eventually will appear in performance parameter measurements.

**Theoretical analysis**

Reflection and transmission coefficients are functions of the constitutive parameters (permittivity, permeability, and conductivity) of the material or medium across the boundary, directions of wave travel (angle of incidence) and the direction of the electric and magnetic fields (wave polarization). Electromagnetic waves propagate in transverse electromagnetic mode (TEM) which is characterized by the absence of longitudinal field in transmissions lines that consist of two or more conductors (co-axial or microstrip lines). Wave propagation with no electric $E$ and magnetic $H$ field components in the direction of propagation is considered at the boundary of two mediums shown in Figure 1d at a certain angle of incidence $\theta_i$ (oblique incidence angle).

**Oblique incident wave** Reflection and transmission coefficients at oblique angle of incidence $\theta_i$ are obtained considering parallel $\parallel$ or perpendicular $\perp$ polarization of electromagnetic waves. For most of the cables and transmission lines dielectric material relative permeability $\mu_i$ is unity. Fresnel equations of reflection coefficient in parallel polarization $\Gamma_\parallel$, transmission
coefficient in perpendicular polarization $\tau_\parallel$, reflection coefficient in perpendicular polarization $\Gamma_\perp$ and transmission coefficient in perpendicular polarization $\tau_\perp$ with $\mu_r = 1$ are shown in Equations 3 - 6, respectively. A detailed explanation of these equations is provided in reference [1]. Subscripts ‘i’, ‘r’ and ‘t’ represent incident, reflected and transmitted fields.

$$\Gamma_\parallel = \frac{E_i}{E_i} = \frac{-\cos \theta_i + \left(\frac{\varepsilon_r}{\varepsilon_i} \sqrt{1 - \left(\frac{\varepsilon_r}{\varepsilon_i}\right)^2 \sin^2 \theta_i}\right)}{\cos \theta_i + \left(\frac{\varepsilon_r}{\varepsilon_i} \sqrt{1 - \left(\frac{\varepsilon_r}{\varepsilon_i}\right)^2 \sin^2 \theta_i}\right)}$$  \hspace{1cm} (Eq. 3)

$$\tau_\parallel = \frac{E_i}{E_i} = \frac{2 \cdot \left(\frac{\varepsilon_r}{\varepsilon_i} \cos \theta_i \sqrt{1 - \left(\frac{\varepsilon_r}{\varepsilon_i}\right)^2 \sin^2 \theta_i}\right)}{\cos \theta_i + \left(\frac{\varepsilon_r}{\varepsilon_i} \sqrt{1 - \left(\frac{\varepsilon_r}{\varepsilon_i}\right)^2 \sin^2 \theta_i}\right)}$$  \hspace{1cm} (Eq. 4)

$$\Gamma_\perp = \frac{E_i}{E_i} = \frac{\cos \theta_i - \left(\frac{\varepsilon_r}{\varepsilon_i} \sqrt{1 - \left(\frac{\varepsilon_r}{\varepsilon_i}\right)^2 \sin^2 \theta_i}\right)}{\cos \theta_i + \left(\frac{\varepsilon_r}{\varepsilon_i} \sqrt{1 - \left(\frac{\varepsilon_r}{\varepsilon_i}\right)^2 \sin^2 \theta_i}\right)}$$  \hspace{1cm} (Eq. 5)

$$\tau_\perp = \frac{E_i}{E_i} = \frac{2 \cdot \cos \theta_i \sqrt{1 - \left(\frac{\varepsilon_r}{\varepsilon_i}\right)^2 \sin^2 \theta_i}}{\cos \theta_i + \left(\frac{\varepsilon_r}{\varepsilon_i} \sqrt{1 - \left(\frac{\varepsilon_r}{\varepsilon_i}\right)^2 \sin^2 \theta_i}\right)}$$  \hspace{1cm} (Eq. 6)

**Normal incident wave**

Wave propagation within two conductor transmission lines is through the length of transmission line that is the normal angle of incidence $\theta_i = 0^\circ$. The Fresnel field reflection and transmission coefficients become polarization independent in the limit as $\theta_i$ goes to zero. Reflection coefficients in Equations 3 and 5 and transmission coefficient in Equations 4 and 6 yield the same results as in Equations 7 and 8. Subscript ‘12’ represents the wave is incident from medium 1 and transmitted into medium 2 of Figure 1d.

$$\Gamma_{12} = \frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$  \hspace{1cm} (Eq. 7)

$$\tau_{12} = \frac{E_t}{E_i} = \frac{2 \cdot \sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} = 1 + \Gamma_{12}$$  \hspace{1cm} (Eq. 8)

**Reflection and transmission coefficients with multiple reflections**

The reflection and transmission coefficients with multiple reflections of dielectric block shown in Figure 1e will be calculated in this section. Figure 2 shows the normal incident plane wave multiple interactions within this dielectric block.
Normal parallel or perpendicularly polarized plane wave incident on the dielectric block can be considered as:

\[ E_i = E_0 e^{j\omega t - \gamma d} \quad \text{(Eq. 9)} \]

Where \( \omega \) is angular frequency, \( d \) is the distance travelled by wave within the dielectric block, and \( \gamma \) is propagation constant of dielectric block and is given in Equation 10. The real part of propagation constant is attenuation constant \( \alpha \) (Np/m) and imaginary is phase constant \( \beta \) (rad/m). \( \varepsilon \), and \( \mu \), in Equation 10 is relative permittivity and permeability of dielectric block (or wave travelling medium).

\[ \gamma = j \frac{\omega}{c} \sqrt{\varepsilon_r \mu_r} = \alpha + j\beta \quad \text{(Eq. 10)} \]

Considering Equation 7 the reflection coefficient of the wave incident from medium 1 into medium 2 and from medium 2 into medium 1 is given in Equations 11 and 12, respectively.

\[ R_{12} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \quad \text{(Eq. 11)} \]

\[ R_{21} = \frac{\sqrt{\varepsilon_2} - \sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} = -R_{12} \quad \text{(Eq. 12)} \]
Considering Equation 8 transmission coefficient for a wave incident from medium 1 into medium 2 and from medium 2 into medium 1 is given in Equations 13 and 14, respectively.

\[
\tau_{12} = \frac{2 \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} = 1 + \Gamma_{12} \quad \text{(Eq. 13)}
\]

\[
\tau_{21} = \frac{2 \sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} = 1 + \Gamma_{21} = 1 - \Gamma_{12} \quad \text{(Eq. 14)}
\]

The total reflected electric field \( E_{rT} \) shown in Figure 4, is equal to the sum of individual reflected electric fields \( (E_{r1}, E_{r2}, E_{r3}, \ldots) \)

\[
E_{rT} = E_{r1} + E_{r2} + E_{r3} + \ldots \quad \text{(Eq. 15)}
\]

where each component is given as:

\[
\begin{align*}
E_{r1} &= E_i \cdot \Gamma_{12} \\
E_{r2} &= E_i \cdot \tau_{12} \cdot \Gamma_{21} \cdot e^{-2 \gamma_d} \\
E_{r3} &= E_i \cdot \tau_{12} \cdot \Gamma_{21} \cdot \Gamma_{21} \cdot \tau_{21} \cdot e^{-4 \gamma_d} \\
E_{r4} &= E_i \cdot \tau_{12} \cdot \Gamma_{21}^{(n-1)} \cdot \Gamma_{21}^{(n-2)} \cdot \tau_{21} \cdot e^{-2 \cdot (n-1) \gamma_d} \\
\end{align*}
\]

(Eq. 16)

Thus, the total reflected field is:

\[
E_{rT} = E_i \left[ \Gamma_{12} + \sum_{n=1}^{\infty} \left( \Gamma_{12} \cdot \tau_{21} \cdot e^{-2 \cdot n \gamma_d} \right) \right] = E_i \left[ \frac{\Gamma_{12} \left( 1 - e^{-2 \gamma_d} \right)}{1 - \Gamma_{12} e^{-2 \gamma_d}} \right] \quad \text{(Eq. 17)}
\]

The total reflection coefficient \( \Gamma_r \) of the dielectric block with multiple reflections is given as:

\[
\Gamma_r = \frac{E_{rT}}{E_i} = E_i \left[ \frac{\Gamma_{12} \left( 1 - e^{-2 \gamma_d} \right)}{1 - \Gamma_{12} e^{-2 \gamma_d}} \right] \quad \text{(Eq. 18)}
\]

Similarly, the total transmitted electric field \( E_{tT} \) shown in Figure 2, is equal to the sum of individually transmitted electric fields \( (E_{t1}, E_{t2}, E_{t3}, \ldots) \)

\[
E_{tT} = E_{t1} + E_{t2} + E_{t3} + \ldots \quad \text{(Eq. 19)}
\]
where, each component is given as:

\[
\begin{align*}
E_{t1} &= E_i \cdot \tau_{12} \cdot \tau_{21} \cdot e^{-\gamma d} \\
E_{t2} &= E_i \cdot \tau_{12} \cdot \Gamma_{21} \cdot \tau_{21} \cdot e^{-3\gamma d} \\
E_{t3} &= E_i \cdot \tau_{12} \cdot \Gamma_{21} \cdot \Gamma_{21} \cdot \tau_{21} \cdot e^{-5\gamma d} \\
E_{tn} &= E_i \cdot \tau_{12} \cdot \Gamma_{21}^{(n-1)} \cdot \tau_{21} \cdot e^{-(2n-1)\gamma d}
\end{align*}
\]  
(Eq. 20)

Thus, the total transmitted field is:

\[
E_{tT} = E_i \cdot \tau_{12} \cdot \tau_{21} \cdot e^{-\gamma d} \sum_{n=0}^{\infty} (\Gamma_{21} \cdot e^{-2\gamma d})^n = E_i \left[ \frac{(1-\Gamma_{21}^2) e^{-\gamma d}}{1-\Gamma_{21}^2 e^{-2\gamma d}} \right]
\]  
(Eq. 21)

Thus, the total transmission coefficient \(\tau_T\) of the dielectric block with multiple reflections is given as:

\[
\tau_T = \frac{E_{tT}}{E_i} = \left[ \frac{(1-\Gamma_{21}^2) e^{-\gamma d}}{1-\Gamma_{21}^2 e^{-2\gamma d}} \right]
\]  
(Eq. 22)

**Understand ripple--Page 4.**
Reflection and transmission coefficients without multiple reflections

Consider a hypothetical scenario without multiple reflections within dielectric block as shown in Figure 3. From Equations 7 and 8 the reflection and transmission coefficient without multiple reflections can be easily written and is shown in Equations 23 and 24. The subscript ‘nr’ represents ‘no reflection’. In some applications a time gating technique is used to remove multiple reflections. Reference [2] explains gating analysis.
Theoretical example

Figure 4 shows plots of: Equation 18 - reflection coefficient with multiple reflections; Equation 22 - transmission coefficient with multiple reflections; Equation 23 - reflection coefficient without multiple reflections; and Equation 24 - transmission coefficient without multiple reflections. For illustration, a dielectric block of relative permittivity ‘10’, length 10 cm, and medium 1 with relative permittivity ‘1’ was considered. Figure 4 illustrates that both reflection and transmission coefficients show ripples only in ‘multiple reflections’ scenario as the result of multiple reflections. However, response ‘without reflections’ does not show any ripples. Multiple reflections can better be viewed considering a time domain response and transmission coefficient time domain response is shown in Figure 5. It can be seen only one large peak (which is due to $E_{t1}$) appears in a ‘without multiple reflections’ scenario. Whereas, in ‘multiple reflections’ case along with the large peak two relatively higher level peaks (which is due to $E_{t2}$ and $E_{t3}$) arise which indicates multiple reflections within the dielectric block.

$$\Gamma_{nr} = \frac{E_{r1}}{E_t} = \frac{\sqrt{\varepsilon_r} - \sqrt{\varepsilon_e}}{\sqrt{\varepsilon_r} + \sqrt{\varepsilon_e}} \quad \text{(Eq. 23)}$$

$$\tau_{nr} = \frac{E_{t1}}{E_t} = \tau_2 \cdot \tau_3 \cdot e^{-\gamma d} = (1 - \Gamma^2_{12})e^{-\gamma d} = (1 - \Gamma^2_{nr})e^{-\gamma d} \quad \text{(Eq. 24)}$$
Simulated observation

Figure 6a shows 50 Ohms microstrip transmission line and Figure 6b shows 30 Ohms shunt resistors deliberately added at input and output of transmission line in order to create mismatch at input and output. In Figure 7 the red line shows the transmission coefficient of 50 Ohms transmission line and is 0 (incident power is completely delivered to load). There are no ripples indicating no reflections. The blue line in Figure 7 shows transmission coefficient of schematic in Figure 6b and is about -12 dB (indicates most of the power is reflected due to mismatch). Also, ripples arise due to multiple reflections within the transmission line.
Figure 6. ADS schematic: (a) 50 Ohms micro strip transmission line; and (b) Mismatch introduced at input and output with 30 Ohms shunt resistors.

Figure 7. Simulated transmission coefficient with and without multiple reflections.

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Experimental verification

Figure 8 is the evaluation board photograph of transmission line simulated in the earlier section. Resistors of 30 Ohms were intentionally placed at input and output SMA connector junctions of the evaluation board (EVM) and is shown in yellow circles. Figure 9 shows measured transmission coefficient of the transmission line in red color superimposed on blue simulated line. The measured data also shows a rippled response due to reflections caused by a mismatch from 30 Ohms resistors placed at the input and output of the transmission line.
Characterization of RF devices sometimes shows ripples in performance parameter measurements such as gain, linearity, return loss, etc. These parameters are directly related to reflection and transmission coefficients. Theoretical analysis supported by simulation and lab measurements was discussed to explain ripple formation in reflection and transmission coefficients. Impedance mismatch causes multiple reflections of electromagnetic waves, which results as ripples.

References

About the author

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