The math of meteorology

Steve Taranovich - February 09, 2013

While I sit here in 30” of snow on Eastern Long Island today, I think about the good, advanced notice we received for this major snow storm and also a few months back for “Sandy”.

Well, on this day, February 9, in 1870 President Ulysses S. Grant signed a congressional resolution “to provide for taking meteorological observations at the military stations in the interior of the continent and at other points in the States and Territories...and for giving notice on the northern [Great] Lakes and on the seacoast by magnetic telegraph and marine signals, of the approach and force of storms.

How does this science predict using mathematics? I found an informative site called Stuff in the Air with some good insights that follow.

Technologies used in meteorology depend greatly on mathematical principles as well as physics. Examples include weather radar, chart usage and interpretation (such as the hodograph shown in Figure 1) and numerical weather prediction.

Figure 1: The hodograph above is one way of representing graphically the vertical pattern or profile of winds measured from a specific location during a certain time.
Meteorologists study the atmosphere. They examine and attempt to predict the weather and the effects of air pollution, amongst other atmospheric wonders.

They also use instruments to measure wind speed and direction, temperature, pressure and humidity. Then, scientific theory helps us understand how these various quantities, sometimes called fields, interact with each other. And how the variables change in time and space.

A mathematical variable is an unspecified quantity able to accept different values under different conditions. In math, we use the letters x and y for most problems. They are symbols representing variables, to which mathematicians use the circumstances of the problem either to determine or assign specific values.

**Meteorology math**

Variables for the mathematics used in meteorology include:

- T for temperature, often qualified with subscripts to denote specific temperatures,
- P for pressure in millibars,
- $\theta$ (a Greek letter, theta), which looks like a zero with a horizontal dash dividing it in half, represents potential temperature. That is, the temperature a package of air would change to if it were suddenly compressed to 1000 millibars without gaining or losing any heat,
- u and v for horizontal velocities of varying types, expressed as vector quantities. Alternatively, they represent vector components of a single velocity and can be combined with w, vertical velocity, to give a three dimensional wind field,
- $\rho$ (the Greek letter rho), which sometimes resembles a backwards 9, to mean density, a function of pressure, temperature and composition,
- the most varying component of composition is humidity. We often use RH for relative humidity and either a lower case r or q to represent the mixing ratio, a measure of absolute humidity.

Because meteorology is a three-dimensional science, four if you include time, the mathematics used in meteorology can require extensive use of partial derivatives. What's that? Partial derivatives allow you to look at how something such as wind speed changes when you move in one direction only, say, straight north. This could be important to pilots. They also let us determine the gradient of a field. That is, to identify what direction to move in order to see the greatest temperature increase, for instance. And even how much it increases after you go a certain distance in that direction.

The mathematics used in meteorology could, and does, fill textbooks quite extensively. A couple of good starter titles are *Atmospheric Science - An Introductory Survey* by Wallace and Hobbs as well as *An Introduction to Dynamic Meteorology* by James R. Holton.

Many of the equations in the texts rely on balancing physical properties, such as in a centripetal force equation. Then each of the forces may be defined by products and derivatives of other parameters, usually. Then they are strung along as terms added together, each a component of the net force in this example. This is not always the case, but often enough.

**Meteorology computers**

What's math without a few computers? *Weather Prediction by Numerical Process* by Lewis Fry Richardson came out in 1922. It said we could simplify the mathematics used in meteorology and, instead of these equations, look at small parametric changes with respect to small physical motions. We could reduce the complex principles to simple algebra.
But it was labor intensive. He also predicted that 64,000 people would be needed to make the calculations needed for predicting the world's weather using this primitive method. Also, his results were quite poor.

Little did he know about computers to be invented just a few decades later. They would really help with the mathematics used in meteorology. With these new machines and a few major refinements in the theory, numerical forecasts became a reality in the early 1950's.

And they have improved, believe it or not, in the several decades following that. We now rely on the models extensively. That's because they have incorporated things like chaos theory and can give ensemble forecasts, which allow for changing probabilities and small statistical variations giving very different results.

It just keeps getting better. Thanks to the mathematics used in meteorology.

**Upper air charts**

Several types of upper air charts give meteorologists and other forecast office personnel shortcuts to explain elaborate principles to users.

What do they do? Meteorology experts check the high elevation weather maps and see many curves bending across from the left side to the right.

![Upper air chart](image)

**Figure 2:** High elevation weather map shown here

Data from the upper air sounding system goes into a computer which then makes these upper air
plots.

What's so special about them? Each curve is a line of equal height above sea-level at which a specified pressure is reached. The label on each upper air analysis map tells us which pressure we're dealing with.

Sometimes many of the curves on upper air charts will lie snugly together and then they will spread out in other areas of the map. The lines loosely represent upper air winds. Several lines together show a well-defined stream and lower intensity upper air currents exist where they spread apart.

In a fashion similar to a river or *gulf stream jet*, these streams meander, merge and separate quite often. And it can look quite random.

![Figure 3: A map of the jet stream separating air masses](image)

**Group kinetics - atmospheric streamlines**

A stream of this sort is a steady upper air *current*. A thousand miles or so long and over a hundred wide, they can last for a moderately long time, at least a few days. You can see these on a forecast jet stream map for instance.

Some *streamlines* span a substantial distance. The largest ones can cover a good portion of a continent and smaller streamlines may only have a regional influence.

We call straight west-east motion in either direction *Zonal Flow*. Meridional flow is quite the opposite. It takes a huge roundabout route, swinging far north and *south* on the map as it makes its way across.

**Air flow research**

An upper air map can show two or more streams simultaneously over the same continent or ocean. Two streams become in-phase when their high and low points, known as ridges and troughs, line up directly above (north and south of) each other. In fact they may appear to merge for a while. We use the term *out of phase* when the big humps are not neatly aligned.

Graphically, a stream shows up on the map as a group of constant *pressure-height* contour lines all huddled together. Why do we care about this arrangement?

One of those lines, dubbed the control line, can signify the entire stream in a summary depiction or
forecast plot. If you want to use ONE line on your weather map to show someone, this is the best one. The ideal curve passes north of a vorticity maximum, or south of a maximum.

![Vorticity Plot](image)

**Figure 3A**: This is vorticity. Think of ceiling fan rotation. We plot it using weather map symbols in units of circle radians per hundred thousand seconds. This time period is a little longer than a day.

For example, many public forecasts show a single line to represent the jet stream. It does not show as a single line on the upper air analyses maps, but the upper air charts show one or several groups of curves that flow together and apart and make any number of jets.

Go back from **Upper Air Charts** to the **Chasing Storms** webpage, or visit the Stuff in the **Air** homepage.

**Temperature calculations in meteorology**

Weather forecasters use a system of conversions to find the average temperature of an upper layer of any thickness. In this way they convert from an individual temperature profile to a typical value for that atmospheric layer.

You can do this on a chart such as a tephigram or skew-t. And you can use layer averaging for temperature conversions or other things such as mixing ratio. But why? Here are some examples of this idea in action:

1) Meteorologists find average temperature to determine the type of precipitation. They may predict rain, snow, hail or other types.

2) Averaging potential temperature enables one to estimate the capacity for atmospheric turbulence near the ground. Especially if it is sunny!

3) Look at the dewpoint curve on one of these charts. If it slopes up to the right, parallel to the mixing ratio lines, at low elevations, it shows that the air mixes well in this layer. A handy idea for forecasting convection and its resulting turbulence or storms.

4) Once again we go back to the good old wet-bulb potential temperature. Yet another one of our temperature conversions. It helps us find temperatures after events such as rain and downbursts. We can also determine what type of air mass the layer belongs to.
Figure 4: Tephigram example is shown here
**How to do it**

First of all, by eyeball. This is a quick, crude technique for temperature conversions. Averaging this way can be effective if the person has sufficient skill.

The formal method goes more like this. Determine what layer you need an average for. Determine what parameter (isotherm for temperature etc) you need to average. Find a value for that parameter, by trial and error if necessary, that satisfies this condition:

You draw a single line or curve along your chosen isopleth (a line or curve of equal values) from the top of your chosen layer to the bottom, so that the area of the "triangle" on one side of the radiosonde (Radiosondes are carried into the atmosphere by weather balloons. While ascending they measure a combination of temperature, humidity, barometric pressure, ozone, and wind velocity and radio the data to a ground system for processing. They are used around the world to gather data for weather predicting.) curve is about equal to the area on the other side. Have a look at this American example...

You can see the equal area technique below.
Figure 6: This graph is an example of the equal area curve

It has the black area above the red line roughly equal to the grey area below. With it we determine that the average temperature between 550 millibars and 650 mbar (3774 metres above sea level and 5076m) was about -5 degrees Celsius. Follow the straight yellow isotherms sloping down to the left, parallel to the black arrow, and note that our average temperature value is halfway between 0 and -10.

As an aside from our temperature conversions instance, notice in this sample drawing the hockey sticks on the right. Many plots have these. They display the wind speeds and directions at the corresponding heights. The balloon’s trajectory, rather than wind meters, gives us this data.

Each long spike means a speed of 10 knots, which is about 11 mph or 19 km/h, while a half spike means 5 and a flag means 50. Additionally, the angle of the shaft shows the directions, with most of these in the picture above displaying westerly or northwest winds with southwest near the surface.

How to deal with humid air? We find vertical profiles of moisture and water vapor maps equally important in examining the effects of dampness on weather. First, we present a recap of the most significant variables used to gauge humidity:
Figure 7: Shown is an NOAA water vapor map

1) **Mixing ratio** \((r)\) mass of water vapour to that of dry air, given in g/kg. Lines of constant mixing ratio, go straight up steeply to the right on a tephigram (See Figure 4).

2) **Relative humidity** \((\text{RH})\) the relative humidity is approximately the ratio of the air’s actual mixing ratio to its mixing ratio if saturated at the same temperature and pressure. Then multiply by a hundred to express as a percent. This one requires data from both temperature and humidity recorders.

Water vapor maps display relative humidity most often. Use the mixing ratio passing through the isobar's intersection with dew point temperature and actual temperature isotherms respectively. If there is no difference between these two temperatures, RH is 100 percent.

3) **Specific humidity** \((q)\) ratio of the mass of water vapor to the total mass, water included.

4) **Absolute humidity** actual density (in g/cubic meter) of the water vapour present in the air.

See also the list of thermodynamic temperature conversions used in tephigram (See Figure 4) analysis.

Meteorologists use thermodynamic charts with names such as the skew-T log-P (See Figure 5) or tephigrams and hodographs (See Figure 1) to interpret the data. They incorporate many standards including the first law thermodynamics lecturers teach us.

An upper isobar weather map can also help in data analysis. They enable people to compare data for
many locations at once.

In many countries, including Canada, forecasters use the tephigram chart (See Figure 4). The area within any polygon on this chart, an isobaric box, is proportional to the energy involved in a physical change. This helps make the application of the first law thermodynamics more visual.

1. Notice the slightly curved lines going left to right, more or less. You will see two labels for each one. Pressure in millibars (a millibar is one tenth of a kilopascal) outside the box, and approximate elevation above sea level, technically called geopotential, in metres just inside the box.

2. The two sets of straight lines, one sloping upwards to the right and the other perpendicular set sloping up to the left, form the backbone of this first law thermodynamics tephi, as they are affectionally called. We call the ones going up to the right the **isotherms**. Each one represents a specific temperature. In this sample chart, you can see one for every 10°C, and more precise tephigrams show one for every single degree.

3. The straight lines inclining to the left stand for the potential temperature we mentioned earlier. These **dry adiabats**, as we know them, have values in Fahrenheit, Celsius, as shown here, or Kelvin, which is Celsius plus about 273. They show the path the temperature of a parcel of dry air would follow if it were descend, go downhill for instance, and and undergo a pressure increase **without gaining or losing any heat or moisture** from the sun, the ground or other places.

The first law thermodynamics courses teach applies here. Look at where each dry adiabat crosses the 1000 mbar pressure curve. A millbar, mbar or mb, is a hundred Pascals. The isotherm crossing that intersection displays the exact same temperature value as the adiabat's potential temperature. That's because we define potential temperature using the 1000 mb pressure value.

4. Now, see the broken curved lines? The label on one says "saturated adiabat". Some scientists call them "pseudo-adiabatic lines". They appear nearly vertical near the bottom of the chart and gradually, asymptotically actually, approach the angle of the dry adiabats near the top. They intersect the 1000 mbar curve at isotherm values known as their web-bulb potential temperatures, used to label these curves. Notice no labels for them appear in the example above. They display the path completely saturated air follows as it goes up the same hill under the same conditions mentioned in the previous illustration.

The graph in Figure 5 makes a viable alternative to the tephigram. The similarities greatly outweigh the differences, and the USA National Weather Service seems to prefer using this one.

The isotherms, moisture content and isobars show as straight lines here while all the others curve gently. But the first law thermodynamics and other laws remain the same.

So there are some examples of the math that goes into weather prediction. As people say, “They are only right 50% of the time”---well, I think it’s a little better than that but it’s certainly better than most of us can do!