Modeling multiconverter DC power systems

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I came across a really good IEEE paper on XPlore regarding a modular approach to model and simulate multiconverter dc power system based on the state-space averaging method.

We have a block diagram showing the interconnecting of converters in multiconverter power electronic systems.

The buck converter

The DC/DC PWM buck converter

For brevity, I will just show the example of a buck converter in continuous conduction mode. For more details on other topologies, please go to IEEE XPlore for the complete paper (Membership is required to IEEE and a fee for IEEE XPlore system---well worth it) The paper goes on to show a
First we define a commutation function $u(t)$:

$$u(t) = \begin{cases} 
1, & 0 < t < dT \\
0, & dT < t < T.
\end{cases}$$

Equation 1

If we apply the commutation function $u(t)$ to the two sets of circuit state-space equations, we get the unified set of circuit state variable equations in continuous conduction mode:

$$\begin{align*}
\frac{di_L}{dt} &= \frac{1}{L} [v_{in}u(t) - v_o] \\
\frac{dv_o}{dt} &= \frac{1}{C} [i_L - i_{out}] \\
i_{in} &= i_L u(t).
\end{align*}$$

Equation 2

The actual state-space variables are the Fourier coefficients of the circuit state variables, $i_L$ and $v_o$. By using the first-order approximation to obtain $i_L$ and $v_o$, we have six real state variables as follows:

$$\begin{align*}
\langle i_L \rangle_1 &= x_1 + jx_2, \quad \langle i_L \rangle_0 = x_5 \\
\langle v_o \rangle_1 &= x_3 + jx_4, \quad \langle v_o \rangle_0 = x_6.
\end{align*}$$

Equation 3

And since $i_L$ and $v_o$ are real:

$$\begin{align*}
\langle i_L \rangle_{-1} &= \langle i_L \rangle_1^*, \quad \langle v_o \rangle_{-1} = \langle v_o \rangle_1^*
\end{align*}$$

Equation 4

Note: The operator * means the conjugate of a complex number.

The circuit state variables are calculated and given by:

$$\begin{align*}
i_L &= x_5 + 2x_1 \cos \omega t - 2x_2 \sin \omega t \\
v_o &= x_6 + 2x_3 \cos \omega t - 2x_4 \sin \omega t.
\end{align*}$$

Equation 5

If we now apply the time-derivative property of the Fourier coefficients in equation 2, and also substitute the Fourier coefficients of the commutation function in equation 1, we get:
\[\langle u(t) \rangle_0 = d, \quad \langle u(t) \rangle_1 = \frac{j}{2\pi}(e^{-j2\pi d} - 1)\]

**Equation 6**

From this result we get equations 7-9 where:

\[
\langle i_{in} \rangle_1 = \left[ dv_1 + \frac{1}{2\pi}\sin(2\pi d)v_5 \right] + j \left[ dv_2 - \frac{1}{2\pi}(1 - \cos(2\pi d))v_5 \right]
\]

**Equation 7**

\[
\langle i_{in} \rangle_0 = \frac{1}{\pi}\sin(2\pi d)v_1 - \frac{1}{\pi}(1 - \cos(2\pi d))v_2 + dv_5
\]

**Equation 8**

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{bmatrix} =
\begin{bmatrix}
0 & -\frac{1}{\pi L} & 0 & 0 & 0 & 0 \\
-\frac{1}{\pi L} & 0 & -\frac{1}{\pi L} & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & \frac{1}{\pi L} & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{\pi L}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
\frac{1}{\pi L}\sin(2\pi d) \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\frac{1}{2\pi L}(1 - \cos(2\pi d))
\]

\[
\begin{bmatrix}
\text{Re}\{\langle i_{in} \rangle_1\} \\
\text{Im}\{\langle i_{in} \rangle_1\} \\
\langle i_{in} \rangle_0
\end{bmatrix}
\]

\[
+ \sum_{j=1}^{N} \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
\frac{1}{\pi L} & 0 & 0 \\
0 & \frac{1}{\pi L} & 0 \\
0 & 0 & -\frac{1}{\pi L}
\end{bmatrix}
\begin{bmatrix}
\text{Re}\{\langle i_{in} \rangle_j \} \\
\text{Im}\{\langle i_{in} \rangle_j \} \\
\langle i_{in} \rangle_j
\end{bmatrix}
\]

**Equation 9**

\(i_{in_j}\) is the input current of the load converter \(\#j\) and \(R\) is the resistive load of the converter.

Equations 7-9 are the generalized state-space averaged model of the buck converter.

**References**