Modeling multiconverter DC power systems

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I came across a really good IEEE paper on XPlore regarding a modular approach to model and simulate multiconverter dc power system based on the state-space averaging method.

![Block diagram showing interconnecting of converters in multiconverter power electronic systems](image)

We have a block diagram showing the interconnecting of converters in multiconverter power electronic systems.

The buck converter

![Buck converter diagram](image)

The DC/DC PWM buck converter

For brevity, I will just show the example of a buck converter in continuous conduction mode. For more details on other topologies, please go to IEEE XPlore for the complete paper (Membership is required to IEEE and a fee for IEEE XPlore system---well worth it) The paper goes on to show a
boost converter and also DC/AC inverters and further analysis.

First we define a commutation function \( u(t) \):

\[
\begin{align*}
    u(t) = & \begin{cases} 
        1, & 0 < t < dT \\
        0, & dT < t < T.
    \end{cases}
\end{align*}
\]

\textbf{Equation 1}

If we apply the commutation function \( u(t) \) to the two sets of circuit state-space equations, we get the unified set of circuit state variable equations in continuous conduction mode:

\[
\begin{align*}
    \frac{di_L}{dt} &= \frac{1}{L} [v_{in}u(t) - v_o] \\
    \frac{dv_o}{dt} &= \frac{1}{C} [i_L - i_{out}] \\
    i_{in} &= i_L u(t).
\end{align*}
\]

\textbf{Equation 2}

The actual state-space variables are the Fourier coefficients of the circuit state variables, \( i_L \) and \( v_o \). By using the first-order approximation to obtain \( i_L \) and \( v_o \), we have six real state variables as follows:

\[
\begin{align*}
    \langle i_L \rangle_1 &= x_1 + jx_2, \langle i_L \rangle_0 = x_5 \\
    \langle v_o \rangle_1 &= x_3 + jx_4, \langle v_o \rangle_0 = x_6.
\end{align*}
\]

\textbf{Equation 3}

And since \( i_L \) and \( v_o \) are real:

\[
\begin{align*}
    \langle i_L \rangle_{-1} &= \langle i_L \rangle^*_1, \langle v_o \rangle_{-1} &= \langle v_o \rangle^*_1
\end{align*}
\]

\textbf{Equation 4}

Note: The operator * means the conjugate of a complex number

The circuit state variables are calculated and given by:

\[
\begin{align*}
    i_L &= x_5 + 2x_1 \cos \omega t - 2x_2 \sin \omega t \\
    v_o &= x_6 + 2x_3 \cos \omega t - 2x_4 \sin \omega t.
\end{align*}
\]

\textbf{Equation 5}

If we now apply the time-derivative property of the Fourier coefficients in equation 2, and also substitute the Fourier coefficients of the commutation function in equation 1, we get:
\[ \langle u(t) \rangle_0 = d, \quad \langle u(t) \rangle_1 = \frac{j}{2\pi} (e^{-j2\pi d} - 1) \]

**Equation 6**

From this result we get equations 7-9 where:

\[ \langle i_{m} \rangle_1 = \left[ \frac{1}{2\pi} \sin(2\pi d) x_5 \right] + j \left[ \frac{1}{2\pi} (1 - \cos(2\pi d)) x_5 \right] \]

**Equation 7**

\[ \langle i_{m} \rangle_0 = \frac{1}{\pi} \sin(2\pi d) x_1 - \frac{1}{\pi} (1 - \cos(2\pi d)) x_2 + dx_5 \]

**Equation 8**

\[
\begin{bmatrix}
    \dot{x}_1 \\
    \dot{x}_2 \\
    \dot{x}_3 \\
    \dot{x}_4 \\
    \dot{x}_5 \\
    \dot{x}_6
\end{bmatrix} =
\begin{bmatrix}
    0 & \omega & -\frac{1}{L} & 0 & 0 & 0 \\
    -\omega & 0 & 0 & -\frac{1}{L} & 0 & 0 \\
    -\frac{1}{C} & 0 & \frac{1}{L} & -\frac{1}{RC} & 0 & 0 \\
    0 & \frac{1}{C} & 0 & -\frac{1}{RC} & 0 & 0 \\
    0 & 0 & 0 & -\frac{1}{RC} & 0 & -\frac{1}{RC} \\
    0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4 \\
    x_5 \\
    x_6
\end{bmatrix} + \begin{bmatrix}
    \frac{d}{L} & 0 & 0 & 0 & 0 & 0 \\
    0 & \frac{d}{L} & 0 & 0 & 0 & 0 \\
    0 & 0 & \frac{d}{L} & 0 & 0 & 0 \\
    0 & 0 & 0 & \frac{d}{L} & 0 & 0 \\
    \frac{1}{\pi L} \sin(2\pi d) & -\frac{1}{\pi L} (1 - \cos(2\pi d)) & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[ \begin{bmatrix}
    \Re\{\langle i_{in} \rangle_1\} \\
    \Im\{\langle i_{in} \rangle_1\} \\
    \Re\{\langle i_{in} \rangle_0\} \\
    \Im\{\langle i_{in} \rangle_0\}
\end{bmatrix} \]

**Equation 9**

\[ i_{in_j} \] is the input current of the load converter \#j and \( R \) is the resistive load of the converter.

Equations 7-9 are the generalized state-space averaged model of the buck converter.

**References**