Pi-Network and "Dipping the Final"

John Dunn - July 15, 2013

An LC circuit often called a "pi-network" is commonly used to match the impedance of a transmitter's load (an antenna) to something more suitable for the transmitter's output amplifier. That output amplifier is very often a beam power vacuum tube whose output impedance, the plate resistance of that tube, is high and therefore well approximated by a current source.

The basic circuit looks like this sketch. The two capacitors, C1 and C2, are shown as variable because we will see how adjusting their respective values affects the pi-network's behavior.

\[
H = \frac{e_2}{i_1} = \frac{R}{s^3L/C_1C_2R + s^2L/C_1 + sR(C_1 + C_2) + 1}
\]

\[
Z_{in} = \frac{e_1}{i_1} = \frac{s^2L/C_2R + sL + R}{s^3L/C_1C_2R + s^2L/C_1 + sR(C_1 + C_2) + 1}
\]

Phase of \(Z_{in} \):

\[
\text{Phase of } Z_{in} = \text{atn} \left( \frac{(WR(C_1 + C_2) - W^3L/C_1C_2R)}{(1 - W^2L/C_1)} \right) - \text{atn} \left( \frac{WL}{R(1 - W^2L/C_2)} \right)
\]

We look at three things. They are 1) the transfer function of the output voltage, \(e_2\), in response to the current excitation, \(i_1\), 2) the impedance, \(Z_{in}\), that is presented to the current source and 3) the phase angle associated with the algebraic expression for \(Z_{in}\).
To serve as an example, let $L = 2 \mu$H and $F = 10$ MHz. When $C_2$ is made large, say 2000 pF, the maximum value of the presented impedance $Z_{in}$ tends to be high. As we adjust $C_1$, we get this kind of result. The peak in $H_{db}$ aligns with the peak in $Z_{in}$ and also aligns with a $180^\circ$ value of the phase function. We see a presented $Z_{in}$ of 11255 Ohms for that value of $C_1$. Let's call this our Light Load.

![Graph showing Light Load](image)

We now reduce the value of capacitor $C_2$ to 1000 pF and again examine the effects of varying $C_1$. As before, the peak in $H_{db}$ aligns with the peak in $Z_{in}$ and also aligns with a $180^\circ$ value of the phase function, but now we see a presented $Z_{in}$ of only 2693 Ohms for that value of $C_1$. We may call this our Moderate Load.

![Graph showing Moderate Load](image)

We now again reduce the value of capacitor $C_2$ to 200 pF and again examine the effects of varying $C_1$. As before, the peak in $H_{db}$ aligns with the peak in $Z_{in}$ and also aligns with a $180^\circ$ value of the phase function, but now we see a presented $Z_{in}$ of only 333 Ohms for that value of $C_1$. We may call this our Full Load.

![Graph showing Full Load](image)
Taking a set of idealized characteristic curves for an assumed vacuum tube amplifier operating in class-C (which now replaces the excitation current source), we see this:

Okay then, all you ham radio operators out there, you've seen this before. C2 is the loading control of your Heathkit DX-20, DX-35, DX-40, DX-60, DX-100, Apache or your Globe Scout 680 or whatever while C1 is the knob that you use to tune the plate current for a "dip" as you go for the highest value of Zin as shown in this analysis.

Derivation of transfer function:
Derivation of impedance equation:

\[ e_1 \left( S^2 L C_2 + \frac{S L}{R} + 1 \right) - e_2 \left( \frac{1}{S L} \right) - \frac{i_1}{L} = 0 \]

\[ e_2 \left( S^2 L C_2 + \frac{S L}{R} + 1 \right) \left( \frac{S C_1 + 1}{S L} \right) - e_2 \left( \frac{1}{S L} \right) - \frac{i_1}{R} = 0 \]

\[ e_2 \left( \left( S^2 L C_2 + \frac{S L}{R} + 1 \right) \left( \frac{S C_1 + 1}{S L} \right) - \left( \frac{1}{S L} \right) \right) = i_1 \]

\[ e_2 \left( S^2 L C_2 + \frac{S L}{R} + 1 \right) \left( S^2 L C_1 + 1 \right) - 1 = i_1 \left( S L \right) \]

\[ H = \frac{e_2}{i_1} = \frac{S L}{\left( S^2 L C_2 + \frac{S L}{R} + 1 \right) \left( S^2 L C_1 + 1 \right) - 1} \]

\[ H = \frac{e_2}{i_1} = \frac{S L}{\left( S^4 L^2 C_1 C_2 + S^3 L^2 C_1 R + S^2 L C_1 + S^2 L C_2 + \frac{S L}{R} + 1 \right)^{-1}} \]

\[ H = \frac{e_2}{i_1} = \frac{R}{S^3 L^2 C_1 C_2 + S^2 L^2 C_1 R + S R \left( C_1 + C_2 \right) + 1} \]
The load "R" will be transformed by the pi-network to a new value as seen by the current source.

\[
Z_{\text{in}} = \frac{1}{\frac{1}{SC2 + \frac{1}{R}}} + \frac{SC1}{\frac{1}{SC2 + \frac{1}{R}} + SL} = \frac{1}{\frac{1}{1 + SRC2} + \frac{SL (1 + SRC2)}{1 + SRC2}} + \frac{SC1}{\frac{1}{1 + SRC2} + \frac{SL (1 + SRC2)}{1 + SRC2}}
\]

\[
Z_{\text{in}} = \frac{1}{\frac{1 + SRC2}{S^2LRC2 + SL + R}} + \frac{SC1}{\frac{1 + SRC2 + S^3RLC1C2 + S^2LC1 + SRC1}{S^2LRC2 + SL + R}}
\]

\[
Z_{\text{in}} = \frac{e1}{i1} = \frac{S^2LC2 R + SL + R}{S^3L C1 C2 R + S^2L C1 + S R (C1 + C2) + 1}
\]

Phase of \( Z_{\text{in}} \):

\[
\text{Phase of } Z_{\text{in}} = \text{atan} \left( \frac{W R (C1 + C2) - W^3 L C1 C2 R}{(1 - W^2 L C1)} \right) - \text{atan} \left( \frac{W L}{R (1 - W^2 L C2)} \right)
\]