Demystifying the RHPZ

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During my academic career I have observed that one of the most difficult subjects to teach as well as to learn is systems theory. All those poles and zeros may make sense in class, but once the student tries to relate them to a physical circuit in the lab, there seems to be a chasm between the blackboard and the bench.

My contention is that the student should be taught first to investigate the circuit intuitively, using as much physical insight as possible, and then to use mathematical tools for a more systematic validation. I will try to illustrate using the much dreaded right half-plane zero (RHPZ) as a vehicle.

Let us start out with the basic circuit of Fig. 1a, consisting of an RC network and an op amp buffer, giving

\[ V_o - V_p = \frac{1}{sC} \frac{1}{R+1/sC} V_i - \frac{1}{1+1/sRC} V_i \]  

(1)

As we know, the value of \( s \) for which the denominator vanishes represents a pole. This value is

\[ s = -\frac{1}{RC} \]  

(2)

Since it is negative, the pole lies in the left half of the \( s \) plane (LHP). The circuit’s response to a 0-t-1 V step, depicted in Fig. 1b, is the familiar exponential transient. (Note that the time scale has been chosen slow enough that we can ignore typical op amp dynamic limitations such as slew rate limiting.)
Suppose we now modify the circuit as in Fig. 2a. What is its step response going to be? Using physical insight we observe that initially, with \( C \) still discharged, we have \( V_p = 0 \), so the op amp will momentarily act as an inverting amplifier with a gain of \(-R_2/R_1\) (= -0.5 V/V with the component values shown.) Consequently, in response to the +1-V input step, \( V_o \) will jump to -0.5 V, as shown in Fig. 2b. However, as \( C \) charges up, \( V_p \) will again produce the exponential transient of Fig. 1b, so we eventually have \( V_p \rightarrow V_i \), in turn implying \( V_o \rightarrow V_i \). Note, incidentally, that the higher the \( R_2/R_1 \) ratio, the bigger the initial negative jump of \( V_o \).

Now is the time to invoke a bit of math to put our intuitive analysis on a more formal foothold. In the circuit of Fig. 2a, \( V_i \) contributes to \( V_o \) through two separate signal paths, namely, the low-frequency path through \( R \), and the high-frequency path through \( R_1 \). Using the superposition principle we thus write

\[
V_o = \left(1 + \frac{R_2}{R_1}\right)V_p - \frac{R_2}{R_1}V_i = \left(\frac{1 + R_2}{1 + sRC} - \frac{R_2}{R_1}\right)V_i
\]

(3)

At low frequencies \((s \rightarrow 0)\) the circuit exhibits a positive gain of +1 V/V, whereas at high frequencies \((s \rightarrow \infty)\) it exhibits a negative gain of \(-R_2/R_1\), so there must be some intermediate frequency at which gain becomes zero. To find the corresponding value of \( s \), aptly called a zero, we let \( V_o = 0 \) in Eq. (3). Solving,

\[
s = \frac{R_1}{R_2} \quad \frac{R_1}{RC}
\]

(4)

We now wish to find the circuit’s transfer function \( H(s) = V_o/V_i \). Expanding Eq. (3) further, and using
Eqs. (2) and (4), we put $H(s)$ in the insightful form

$$H(s) = \frac{1 - s/\omega_z}{1 + s/\omega_p}$$

(5)

where $\omega_z$ is called the zero frequency and $\omega_p$ the pole-frequency,

$$\omega_p = \frac{1}{RC} \quad \omega_z = \frac{R_1}{R_2} \omega_p$$

(6)

It is good practice to check that Eq. (5) is consistent with our intuitive conclusions. Indeed, $H \to 1$ for $s \to 0$, $H = 0$ for $s = \omega_z$, and $H = -\omega_z/\omega_p = -R_2/R_1$ for $s \to \infty$. The pole-zero plot of $H(s)$, shown in Fig. 3, indicates that the circuit exhibits a left half-plane pole (LHPP) as well as a right half-plane zero (RHPZ).

![Figure 3 - Complex-plane pole-zero plot for the circuit of Fig. 2.](image)

**What do we make of all this?**

A RHPZ is of potential concern in negative-feedback systems, two popular examples of which are the RHPZ arising in the peak-current-mode control of boost switching regulators, and the RHPZ arising in Miller-type frequency compensation of op amps, especially two-stage CMOS op amps.

Consider first the **boost regulator case**. Should the loop controller issue a command in the form of a voltage step, the RHPZ will respond, at least initially, in the opposite direction, as per Fig. 2b. If the controller is fast enough, it will overreact by reversing the polarity of its own initial command, only to meet again a reaction in the opposite direction. It appears as if feedback turned, at least momentarily, from negative to positive, which is a recipe for possible oscillation.

A common cure is to deliberately slow down the loop controller so as to prevent it from over-reacting to the RHPZ. Sure enough, if we change its input from an abrupt step to a gradual ramp, the circuit of Fig. 2a will respond in the intended direction almost right from the beginning, as demonstrated in Fig. 4 (for convenience, also $V_p$ is shown.) In frequency-compensation parlance [1], slowing down the loop controller means making the loop’s crossover frequency $\omega_x$ sufficiently lower than $\omega_z$, so there isn’t enough loop gain to cause significant overreaction to the RHPZ (perhaps a future blog on this is in order.)
Next, let us turn to the Miller-compensation case, which is more conveniently investigated in the frequency domain. Letting $s \to j\omega$ in Eq. (5) we find the phase response as

$$\phi H = \tan^{-1}(-\omega/\omega_z) - \tan^{-1}(\omega/\omega_z) - \tan^{-1}(\omega/\omega_x) - \tan^{-1}(\omega/\omega_y)$$

indicating that the RHPZ contributes phase delay just like the LHPP does. So, while the LHPP of Fig. 1a contributes up to -90° of delay, the LHPP-RHPZ of Fig. 2a contributes up to -180° of delay, again a recipe for possible oscillation. (When used with $R_2 = R_1$, the circuit finds application as a phase shifter.)

In bipolar op amps the RHPZ is usually much higher than the loop’s crossover frequency $\omega_x$, so it is of no concern. Not so in multi-stage CMOS op amps, where $\omega_z$ may be close to or even lower than $\omega_x$ (this is due to the notoriously low transconductance $g_m$ of FETs). There are various ways of coping with this [2], including pushing $\omega_z$ well out of the way, such as $\omega_z \to \infty$.

In the circuit of Fig. 2 you can make $\omega_z \to \infty$ either by letting $R_2 \to 0$, or $R_1 \to \infty$, or both (all trivial cases). But what if, for some reason, you are not allowed to touch the $R_1-R_2$ network? Then, turn the op amp into a difference amplifier so that it treats $V_i$ as a common-mode input, and therefore responds to the input step with $V_o = 0$, at least initially. To this end you need to insert a “compensation” resistance $R_c (= RR_2/R_1 = 0.5 \text{ kΩ})$, as shown in Fig. 5a. This will also change $\omega_p$ and, hence, $V_p$ (see Fig. 5b, and compare with Fig. 1b), but at least you have the satisfaction of having eliminated the RHPZ (perhaps a future blog on this is in order.)
An analogy.
During my graduate-student days I bought my first car, a 1955 Ford Fairlane, for $50 “as is” (or, rather, “as was”). Once I took possession, I immediately hit the highway to experience the excitement of pressing the gas pedal all the way to the floor… only to discover that the engine reacted almost as if it were suffocating. Only after some unnerving initial hesitation would the car adjust itself and gradually pick up speed. My engine must have had a RHPZ!

Soon I found empirically that the car would respond much better (no suffocation) if I pushed the pedal gradually instead of abruptly. I mentioned this to a musician friend of mine who was making a living fixing and reselling junkyard cars. He disappeared under the hood and magically restored the engine to mint condition. I never bothered to ask what he did, but I guess he must have sent my RHPZ to infinity…

References