The basics of digital signal spectra

Daniel Chow - October 15, 2013

When designing with interconnects, we're often concerned about issues such as crosstalk and reflections. We frequently ask for models from the manufacturers so we can run simulations to determine the quality of these passive components. The outputs of these simulations are typically eye diagrams, which are a good qualitative indicator of signal integrity. The frequency spectra of the signals are, however, often overlooked, especially if they're complex data patterns.

Before jumping into complex data patterns, I'll start with a single square pulse, which is the fundamental building block of binary NRZ (non-return-to-zero) signals. Figure 1 shows a plot of a single square pulse of width $T_0$ and the corresponding Fourier spectrum, where $f_0$ is defined as $1/T_0$. We see that the magnitude spectrum is a rectified sinc function, i.e., $\text{sinc}(x) = \sin(x)/x$. The spectrum is continuous, which means it requires an infinite number of frequency components to perfectly reproduce.
Figure 1. A single digital pulse and its corresponding frequency spectrum.

Next, consider a square wave, which is simply a chain of square pulses. Figure 2 shows the plot of a square wave and the corresponding spectra. A square wave consists of the fundamental frequency plus only the odd harmonics, which diminish in magnitude as \(1/f\). For consistency, the width of a square pulse is still \(T_0\). We see that the spectral lines are located at \(0.5f_0\), \(1.5f_0\), and \(2.5f_0\). I also plot the sinc function from the square pulse as comparison. We see that the spectral lines of the square wave align with the peaks of the sinc function.
Figure 2. A square wave's frequency spectrum consists of the fundamental frequency and odd harmonics only.

Now, let’s make things interesting by building up a short data pattern. In this case, I use the k28.5 pattern. **Figure 3** shows the waveform of this pattern. As before, the width of a single square pulse is $T_0$. 
This is where things get interesting. We see that the spectrum for k28.5 consists of many lines, all of which fall under the envelope of the sinc function. The lowest frequency spectral line is $f_0/20$, which we define as $f_{\text{PAT}}$ and is indicative of the fact that k28.5 is 20 bits long. Interestingly, all other spectral lines are odd harmonics of $f_{\text{PAT}}$. This is because k28.5 consists of two identical parts, only differing in disparity, which creates an odd symmetry.

The locations of the spectral lines would be the same for any 20-bit long pattern. What makes this spectrum unique to k28.5 is the magnitude of the lines along with their relative phases. Looking back at the square wave case, we see that a square wave is also a 2-bit long pattern of alternating 1s and 0s with odd symmetry. The pattern repeat frequency is $f_0/2$, which is exactly why we only see the fundamental frequency of $0.5f_0$ and odd harmonics.

Now look at PRBS $2^7-1$, which is a longer pattern. Following the concepts introduced in the previous cases, we expect to see a spectrum of dense lines starting with $f_0/127$, all of which fall under the sinc envelope. Since PRBS has no symmetry, we expect to see all harmonics of $f_0/127$. **Figure 4** shows the waveform and spectrum of this signal with behavior matching our expectations.
Figure 4. The frequency spectrum on an infinitely log data pattern is infinitely dense, but still within the sinc envelope.

For an infinitely long, non-repeating pattern, you would expect $f_{\text{EXT}} = f_0/\infty$, which means that the spectrum is infinitely dense, i.e., continuous. Of course, the lines still have to fit under the sinc envelope.

Looking back to the case of a simple square pulse, we now see it for what it really is: an infinitely long chain of zeros interrupted by a single one, which explains why the spectrum is a continuous, single function. This is exactly the sort of elegance that I love in mathematics, and we easily see that it fits into our applications in signal integrity.

So far, we've discussed the spectra of binary signals. In capacitatively-coupled or inductively-coupled crosstalk, however we expect only the $dV/dt$ or $dI/dt$ of the aggressors to impact the victim. I'll discuss this on page 2.

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In crosstalk situations, however, the coupling mechanisms are mostly sensitive to the rising and falling edges, i.e., $dV/dt$ or $dI/dt$. As before, let's start with the basic unit of a square pulse. Taking
the time derivative (i.e., $dV/dt$ or $dI/dt$), we get a positive delta function for the rising edge and a negative delta function for the falling edge. Figure 5 shows the original square pulse (in grey) and the corresponding delta functions. As before, the width of the square pulse is $T_0$.

![Figure 5](image)

**Figure 5.** Delta functions have considerably high frequency content with positive and negative peaks.

The Fourier spectrum of two delta functions is given in the lower half of Figure 5. We see that it is a rectified sine function with zeros at multiples of $f_0 = 1/T_0$. For comparison, the sinc function-shaped spectrum of a square pulse is plotted in gray.

The plots tell us that the spectrum of two delta functions require equal contributions from high and low frequency components. Intuitively, this makes sense as an infinitely sharp delta function has strong high frequency behavior.

Next, consider a square wave, whose derivative is a chain of delta functions alternating in sign. In Figure 6, you can see that the corresponding spectra consists of only odd harmonics of $0.5f_0$, similar to the square-wave case. The only difference is that the magnitude of the lines doesn't diminish with increasing frequency because delta functions have strong high-frequency content. Also, the location of the line aligns with the peaks of the rectified sine function (shown in grey).
Figure 6. Like square waves, a series of delta functions has only odd harmonics of $0.5f_0$.

Moving on to the case of a short pattern (k28.5, 20 bits long), Figure 7 shows us that the spectrum consists of discrete lines starting at $f_{pat} = f_0/20$ plus odd harmonics. Again, the lines all neatly fit under the rectified sine function. The major difference between the square signals and the delta functions is that the delta functions have stronger high-frequency content.
Figure 7. In this 20-bit pattern, the spectral lines fit under the curve.

Continuing with a longer pattern (PRBS $2^7-1$, 127 bits long), we expect to see dense spectral lines spaced by $f_{\text{PAT}} = f_0/127$, all neatly fitting in the envelope of the rectified sine function. **Figure 8** confirms this expectation!
Figure 8. In a 127 bit PRBS7 pattern, the spectral lines are closer together and look continuous compared to the 20-bit pattern from Figure 7.

Comparing what I showed today with the previous blog post for binary data, you can see that the frequencies of the spectral lines depend only on the pattern. The magnitude envelope of the spectral lines depends only on the fundamental component of the signal (i.e., square pulse or delta functions).

These examples help us build an intuitive feel for what a crosstalk aggressors look like if the coupling mechanism solely depended on dV/dt or dI/dt. In most realistic crosstalk cases, however, it’s never quite so simple, which I’ll discuss on page 3.

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Here on page 3, I’ll finish by looking at a case of capacitive coupling such that a square pulse results in two exponential decays for each transition edge, as shown below. I chose this case because the Fourier transform of an exponential decay is an analytical function known as a Lorentzian (Figure 9).
The spectrum for the capacitively coupled square pulse is the product of a rectified sine function and a Lorentzian (shown in red in the image above). The Lorentzian is shown in gray as a reference. Comparing this result with the corresponding cases in the previous two blog posts, we see that it exhibits some similar characteristics.

As was the case with the spectrum for the square pulse, high frequencies have less contribution. Furthermore, like the spectrum for dV/dt of the square pulse, there is no energy at DC. Most importantly, all cases show nulls at multiples of $f_0$.

Just as we did before, we look at a capacitively coupled square wave, shown below. As with all the previous corresponding cases, we get only discrete spectral lines at $0.5f_0$ and odd harmonics. In this case, the magnitude of the spectral lines fits under the Lorentzian-rectified sine envelope -- except where I have roundoff errors in my calculations (Figure 10).
Figure 10. Discrete spectral lines at 0.5f₀ and odd harmonics.

Moving on to a capacitively coupled k28.5 pattern, we see below that there are discrete spectral lines at the same frequencies as the corresponding cases in the previous posts, consisting of f_{PAT} = f₀/20 and odd harmonics. As we should expect by now, though, the spectral lines fit under the Lorentzian-rectified sine envelope (Figure 11).
Lastly, we look at the case for PRBS $2^7-1$. We now have a strong intuition about what the spectrum looks like. The spectrum shown below consists of dense discrete lines starting at $f_{\text{Pat}} = f_0/127$ and multiple harmonics, all of which fit under the Lorentzian-rectified sine envelope (Figure 12).
I've shown you signal spectra for different types of patterns and coupling mechanisms. The spectral shape of the basic unit of the signal (i.e., square pulse, dV/dt, exponential decays) dictates the overall envelope of the signal spectrum, regardless of pattern. The frequencies of the spectral components depend on the length of the pattern. For any repeating pattern, the spectrum consists of discrete lines spaced by the pattern repetition frequency, $f_{\text{PAT}}$.

For the simplest case of a two-bit pattern, the lines are at $f_0/2$ and odd harmonics. For a nonrepeating pattern (i.e., infinite pattern length), the line spacing becomes infinitesimally small, resulting in a continuous spectrum. With these simple guidelines, you can qualitatively estimate the spectrum for any data signal.

We're accustomed to seeing data signals in the time domain, but most signal integrity analysis requires manipulations in the frequency domain -- channel loss, reflections, crosstalk, etc. Therefore, it's important to recognize the spectral properties of data signals.

Daniel Chow's Profile

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