Find the resistance of a long ladder

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In Resistor combinations: How many values using 1kohm resistors?, Glen Chenier showed us how many combinations of resistor combinations you can get. Many people commented, trying to develop a formula for series and parallel combinations. Along those same lines, I present a puzzle asking how do you find the equivalent resistance of resistive "ladders."

An example of a resistive "ladder" is in Figure 1. You can consider it also a cascade of equal cells, each consisting of a series resistor $R_1$ and a transversal resistor $R_2$. Such ladders of R-2R type are widely used in a kind of D/A converters. An n-bit D/A converter of this type contain an n-cell ladder. Theoretically, however the "length" is not limited and can even be infinite.

The resistance $R_{AB}$ at A, B input terminals has a definite value even in such case.

It can be determined from a simple thought, illustrated in Figure 2. Here the whole network, except of cell 1 is replaced by a resistance $R_{CD}$, representing an input resistance of a ladder consisting of cells 2,3, and the rest. Terminals denoted as C, D can be considered “input” terminals of this “depleted” ladder. Also this depleted ladder has infinite length and therefore it holds true that $R_{AB} = R_{CD}$

From Figure 2, you can easily derive:
\[
R_{AB} = R_1 + R_2 \quad | \quad R_{AB} = R_1 + \frac{R_2 \times R_{AB}}{R_2 + R_{AB}} \quad (1)
\]

By solving Equation 1, you get:

\[
R_{AB} = \frac{R_1}{2} \left( 1 + \sqrt{1 + \frac{4R_2}{R_1}} \right) \quad (2)
\]

If for example, \(R_2/R_1=2\), then you get \(R_{AB} = 2R_1\).

What is the input resistance of a ladder comprising \(n\)-cells as in Figure 1, when you terminate it with a resistor of the value

\[
R_{\text{term}} = \frac{R_1}{2} \left( 1 + \sqrt{1 + \frac{4R_2}{R_1}} \right)
\]

Also see:

- Resistors aren't resistors
- Resistor datasheets: Reading between the lines
- Resistor combinations: How many values using 1kohm resistors?
- Resistor 101
- Make schematic symbols understandable
- Transcendental resistors simplify precision design