Picking up from my introductory blog on negative feedback [1], I now wish to discuss its two most common op amp implementations, the inverting and noninverting configurations. These popular circuits exhibit similarities/differences, some tricky, that I will try to clarify.

Let’s first reexamine the block diagram of Fig. 1a in more detail. Writing

\[ V_o = \alpha E = \alpha \left( V_I - \frac{V_o}{A_{\text{ideal}}} \right) \]  

expanding, collecting, and solving for the ratio \( V_o/V_I \), which we shall call the closed-loop gain \( A \), gives

\[ A = \frac{V_o}{V_I} = A_{\text{ideal}} \frac{1}{1 + A_{\text{ideal}}/\alpha} = A_{\text{ideal}} \frac{1}{1 + 1/T} \]
where

\[ T = \frac{a_e}{A_{\text{ideal}}} \]  

We observe that as the error signal \( v_e \) propagates around the loop, it is first amplified by \( a_e \), then is attenuated to \( 1/A_{\text{ideal}} \), and finally is inverted at the summer, for an overall gain of \(-a_e/A_{\text{ideal}}\). Its negative is called – if improperly – the loop gain \( T \). As we move along, we’ll see that \( T \) provides all relevant information about a negative-feedback circuit, so it pays to think of \( T \) as the circuit’s DNA.

For large \( T \) (that is, for \( a_e >> A_{\text{ideal}} \)) we can approximate Eq. (2) as

\[ A = A_{\text{ideal}} \left( 1 - \frac{1}{T} \right) \]  

so here is the first piece of DNA information: \( T \) tells you how close your actual gain \( A \) is to \( A_{\text{ideal}} \). You want a gain error not exceeding 0.1%? Then you need \( T \geq 10^3 \), which you achieve by using an error amplifier with \( a_e \geq 10^3 A_{\text{ideal}} \). Equivalently, \( T \) tells you how much open-loop gain \( a_e \) you need to “throw away” in order to approach \( A_{\text{ideal}} \) within a given percentage error. (All this is familiar stuff, but I wanted to put it down explicitly to prepare the framework for the rest of this blog and the blogs to follow.)

The Noninverting Configuration

The noninverting amplifier of Figure 1b fits the block diagram of Figure 1a to a \( T \) (pun intended). The idea here is that if you want to set it up for a given \( A_{\text{ideal}} \), you use a voltage divider such that

\[ \frac{1}{A_{\text{ideal}}} = \frac{R_2}{R_1 + R_2} = \frac{1}{1 + R_2/R_1} \]  

The combined role of input summer and error amplifier is played by the op amp, whose voltage gain we denote as \( a_e \), and error signal as \( v_e \). Recycling Eq. (2), we thus write

\[ A = \frac{v_o}{v_i} = \left( 1 + \frac{R_2}{R_1} \right) \frac{1}{1 + (1 + R_2/R_1)/a_e} = \left( 1 + \frac{R_2}{R_1} \right) \frac{1}{1 + 1/T} \]  

(6)
indicating a loop gain of $T = a/(1 + R_2/R_1)$.

**The Inverting Configuration**

Let us now turn to the noninverting amplifier’s closest sibling, the *inverting amplifier* of Figure 2a. This configuration does not conform to the block diagram of Fig. 1a as obviously as the noninverting configuration, but we can still analyze it by applying the superposition principle,

$$v_o = a_v (-v_d) = -a_v \left( \frac{R_2}{R_1 + R_2} v_I + \frac{R_1}{R_1 + R_2} v_o \right)$$

Again expanding, collecting, and solving for the *closed-loop gain*, we get

$$A = \frac{v_o}{v_I} = \left( -\frac{R_2}{R_1} \right) \frac{1}{1 + (R_2/R_1)/a_v} = \left( -\frac{R_2}{R_1} \right) \frac{1}{1 + 1/T}$$

**Figure 2** - *(a)* The inverting amplifier, and *(b)* source transformation to show it in its most natural form.

The loop gain is still $T = a/(1 + R_2/R_1)$, and this makes sense because as $v_o$ propagates around the loop, it still undergoes magnification by $a_v$ and then attenuation to $1/(1 + R_2/R_1)$. However, we now have $A_{\text{ideal}} = -R_2/R_1$, so we can no longer claim that $T = a_v/A_{\text{ideal}}$. What gives? This is the first potential source of *confusion* between the inverting and noninverting configurations, which students have rightly pointed out in my classes countless times.

The confusion stems from the fact that even though we are driving the circuit with a voltage, its natural input is really a current [2], as evidenced by the source transformation of Figure 2b. Nobody in his or her right mind would drive an op amp’s inverting input pin with a voltage source, as this would prevent the op amp from exercising its negative-feedback action. So, we decouple $v_o$ from $v_I$ by means of $R_1$, and in so doing we convert $v_I$ to $i_I (= v_I/R_1)$, making the op amp act as a
current-to-voltage (I-V) converter. This, even though the op amp is inherently a voltage-to-voltage (V-V) converter. Replacing $v_i$ with $R_1i$ and manipulating, we put Equation (7) in the form of Equation (1)

$$v_o = -a_v \left( \frac{R_1}{R_2} \right) \left( i_i - \frac{1}{-R_2} v_o \right)$$  \hspace{1cm} (9)

Comparison with Equation (1) indicates that the circuit realizes a negative-feedback system with

$$a_v = -a_v \left( \frac{R_1}{R_2} \right) \quad A_{v(\text{ideal})} = \lim_{i_i \to 0} \frac{v_o}{i_i} = -R_2 \quad T = \frac{a_v}{1 + R_2 / R_1}$$  \hspace{1cm} (10)

where both the error gain $a_v (\neq a_v!)$ and the I-V converter gain $A_{iv}$ are in V/A. The ideal voltage gain from $v_i$ and $v_o$ is then $A_{v(\text{ideal})} = v_o / v_i = (v_o / i_i) \times (i_i / v_i) = A_{iv(\text{ideal})} / (R_i) = -R_2 / R_1$, thus confirming Equation (8).

Summarizing, while sharing the same expression for $T$, the inverting and noninverting amplifiers differ both in the magnitudes and polarities of their voltage gains.

The quantity $1 + R_2 / R_1$, common to both amplifiers, is called the noise gain because this is the gain with which either circuit will amplify any noise present between the op amp’s input pins, such as the input offset voltage $V_{o,s}$. Noise gain and signal gain coincide in the noninverting configuration, but differ in the inverting configuration, as the latter’s signal gain is $-R_2 / R_1$.

Comparing the Frequency Responses

Most op amps are designed for a frequency response $A_v(jf)$ of the type of Fig. 3. Gain starts out high and then rolls off with frequency until it drops to unity, or 0-dB, at a frequency aptly called the transition frequency $f_t$. We wonder how this affects the closed-loop gain $A(jf)$. For both amplifiers this gain takes on the common form

$$A(jf) = A_{v(\text{ideal})} D(jf)$$  \hspace{1cm} (11)

where
is the discrepancy function, so called because it tells you by how much your actual gain departs from the ideal. Since $T(jf) = a_v(jf)/(1 + R_2/R_1)$, you can visualize the decibel plot of $|T|$ as the difference between the plot of $|a_v|$ and the noise-gain curve $1 + R_2/R_1$ (this, because the log of a ratio equals the difference of the logs). As depicted in Figure 3, $|T|$ is large at low frequencies, making $|D|$ approach 0 dB, or unity there. However, $|T|$ itself rolls off with frequency until it drops to 0 dB, or unity, at the crossover frequency

$$f_x = \frac{f_c}{1 + R_2/R_1}$$

(13)

The phase angle of $a_v$ at $f_c$ is close to $-90^\circ$, so we can write $T(jf_c) \approx 1 \angle -90^\circ = -j$. Then, $D(jf_c) = 1/[1 + 1/(1+j)] = 1/(1+j)$, so $|D(jf_c)| = 1/\sqrt{2} = -3$ dB. Clearly, the crossover frequency $f_c$ represents the closed-loop bandwidth (this is the second important piece of DNA information). Above $f_c$, $|D|$ itself rolls off with frequency, so $D(jf)$ is just the ordinary low-pass function with $f_c$ as the -3-dB frequency,

$$D(jf) = \frac{1}{1 + 1/jf / f_c}$$

(14)

With identical $R_2/R_1$ ratios, the inverting and noninverting amplifiers exhibit the same noise gain and closed-loop bandwidth, but dc signal gains of $-R_2/R_1$ and $1 + R_2/R_1$, respectively, so

$$A_{invi}(jf) = \frac{1 + R_2/R_1}{1 + 1/jf / f_c} \quad A_{inv}(jf) = \frac{-R_2/R_1}{1 + 1/jf / f_c}$$

(15)

The difference in their magnitudes is minor for high $R_2/R_1$ ratios, but becomes more pronounced at
low gains. The greatest difference occurs when both amplifiers are configured for unity gain, as depicted in Figure 4. In this case we have $1 + R_2/R_1 = 1 + 0/\infty = 1$ for the follower, so $f_c = f_v$, but $1 + R_2/R_1 = 1 + 1 = 2$ (or 6 dB) for the inverter, so $f_c = f_v/2$. Clearly, the -3-dB frequency of the inverter is half as large as that of the follower.

The two circuits exhibit also a marked difference in the input resistance $R_i$. So long as $v_o$ is vanishingly small, the current drawn from the input source in Fig. 1b will also be vanishingly small (this current is $v_o/r_d$, $r_d$ being the op amp’s internal input resistance, usually large). Consequently, the noninverting amplifier presents a very high input resistance $R_i$ (ideally $R_i \to \infty$), thus eliminating loading at the input (in fact, this is the very reason for using the follower as a buffer). In Fig. 2a, however, a vanishingly small $v_o$ will make the inverting-input node approach a virtual ground, so this circuit presents a non-infinite input resistance of $R_i = R_o$, which is likely to cause input loading.

![Figure 4](image)

**Figure 4 - Comparing unity-gain (a) the voltage follower and (b) inverting amplifier.**

Compared with the voltage follower, the unity-gain inverter has (a) lower input resistance, (b) half as much bandwidth, (c) twice as much noise gain, and (d) it even requires two resistors (compared to the follower’s plain feedback wire) to achieve less. So, why use it at all? Your answer here:__________.

References


[2] [http://online.sfsu.edu/sfranco/BookAnalog/AnalogJacket.pdf](http://online.sfsu.edu/sfranco/BookAnalog/AnalogJacket.pdf)