Three-op-amp state-variable filter perfects the notch

Alain Temps - December 15, 2014

The usual schematic of a state-variable filter with two inverting integrators is well known.

Curiously, the input signal is almost always connected to the minus input of U1. Figure 1 is an example with $f_0 = 1$kHz and $Q = 5$.

![Figure 1 Typical state-variable filter](image)

The circuit is known for its versatility, and ability to simultaneously provide low-pass, band-pass, and high-pass outputs. Gain, center frequency, and Q may be adjusted separately. A notch filter is usually obtained by adding a fourth op-amp, either to sum the LP & HP outputs (which are out of phase), or to difference the input and BP outputs (which are in-phase). The notch depth then depends on the matching of the resistances used for adding or subtracting the signals.

In this Design Idea, the input signal is instead connected to the positive input of U1; the filter naturally generates two notch outputs, without the need to combine any ports.
These notch outputs are taken from the two inputs of U1, labeled V1 & V2. They are: the sum of the input and BP output for V1, and the sum of the HP & LP outputs for V2.

The complete equation is:

\[ \frac{V_1}{V_e} = \frac{R_{15}}{R_{14} + R_{15}} \left[ 1 - \frac{R_1 C_1 R_2 C_2 R_{13}}{R_{12}} \right] / \left[ 1 + j \frac{R_2 C_2 R_{14}}{R_{14} + R_{15}} \frac{R_{13}}{R_{12}} - \frac{R_1 C_1 R_2 C_2 R_{13}}{R_{12}} \right] \]

where \( R_{123} = R_{11} || R_{12} || R_{13} \)

The numerator always has an exact zero at \( \phi = 1 / v \left( R_1 C_1 R_2 C_2 \frac{R_{13}}{R_{12}} \right) \).
Low frequency gain is always equal to high frequency gain, which means that rejection is naturally infinite at the center frequency and does not depend on component tolerances. Amongst all notch filters, only the Bainter filter (and here) also possesses this property, but its parameters cannot easily be tuned separately.

Further equations:

$$Q_D = \left(1 + \frac{R_{15}}{R_{14}}\right) \frac{v(R_1C_1/R_2C_2)}{\left[ v(R_{12}R_{13})/R_{11} + v(R_{12}R_{13}) + v(R_{12}R_{12})\right]}$$

$Q_D$ is maximum, and the equations greatly simplify, if we choose $R_{12} = R_{13}$. Then:

$$V_2/V_1 = \frac{R_{12}/(R_{14}+R_{13})}{1 - ?^2 R_1C_1R_2C_2} \left[ 1 + j? R_2C_2 (2+R_{12}/R_{11})/(1+R_{13}/R_{14}) - ?^2 R_1C_1R_2C_2 \right]$$

LF & HF gain: $A_0 = \frac{R_{15}}{(R_{14}+R_{15})}$

Notch frequency: $?_0 = 1/v(R_1C_1R_2C_2)$ - may be tuned with $R_1$ & $R_2$

Q: $Q_0 = (1+R_{13}/R_{14})/(2+R_{12}/R_{11}) \frac{v(R_1C_1/R_2C_2)}{\left[ v(R_{12}R_{13})/R_{11}\right]}$ - may be tuned with $R_{11}$

Practically, simulation shows that notch rejection is better on $V_2$ than on $V_1$. It may exceed 80dB with high speed op-amps at U2 & U3, and is limited by the op-amp specs.

Input impedance is not constant with frequency. However, neither rejection depth nor gain depend on source resistance, which appears in series with $R_{14}$ and slightly decreases gain and $Q_0$ (it is the same for Figure 1).

An optional buffer U4 with gain equal to $(1+ R_{16}/R_{14})$ may isolate the filter from any external disturbance, and maintain gain at +1. The overall gain may of course be adjusted with $R_3$ or $R_4$.

The output noise is extremely low across the whole spectrum; even lower at $?_o$.

For example, for U1, U2, & U3 with $e_n = 5$ nV/vHz, total noise at $V_2 = 4.5$ nV/vHz @ $?_o$ and 6.4 nV/vHz in the rest of the spectrum.
Figure 4 Noise vs. frequency

Be wary of possible saturation of U1 to U3 at center frequency $\omega_0$ because their gains are high if Q is high, as with any state variable filter.

The gains of U1-U3 at $\omega_0$ are:

U1: $-\frac{R_{12}}{R_{14}} v\left(\frac{R_1C_1}{R_2C_2}\right)$

U2: $\frac{R_{13}}{R_{14}}$

U3: $\frac{2+R_{12}/R_{11}}{2(1+R_{14}/R_{13})}$

Saturation characteristics may be improved by increasing the ratio $C_1/C_2$, and/or decreasing $R_{13}/R_{14}$, at the expense of higher noise.

A simulation file can be downloaded.

Also see:

- Notch filter is insensitive to component tolerances
- Filter quashes 60-Hz interference