What is the resonant frequency of a cavity?:
Rule of Thumb #30

Eric Bogatin - July 19, 2016

Signal paths are designed to be low Q resonators. This prevents ringing. But cavities, composed of a power and ground plane for example, can have very high Qs. This means even a little bit of coupling from signal paths can drive resonances and give rise to long range noise voltages within the cavity. Knowing the cavity resonant frequencies you might expect to see can warn of potential problems while you can still fix them.

**Spoiler summary:** When the cavity has open ends (boundary conditions), resonances arise when an integral multiple of half-wavelengths can fit between the ends of the cavity. In a cavity of length $Len$, filled with an FR4-like material, the resonant frequencies are:

$$f_{res}[GHz] = \frac{n \cdot 7.5}{Len[inch]} = \frac{n \cdot 3}{Len[cm]}$$

**Remember:** before you start using rules of thumb, be sure to read the Rule of Thumb #0: Using rules of thumb wisely.

**Previous:** Rule of Thumb #29: The spatial extent of the rising edge

When an interconnect has very large impedance discontinuities at its ends - like when it is open - and a signal is leaked into it, the signal will reflect off each end and rattle around. At some frequencies, each reflection is in phase with the new incident wave and the multiple waves build up. These frequencies are resonances.

The analysis of resonances is most easily done in the frequency domain. We launch a sine wave signal into one end of the cavity. It travels down to and then reflects from the far end, returning to the launch source.

When it reflects off the open far end, there is no phase change. The wave then propagates to the launch end and reflects again, with no phase change. The round trip time is:
Where $c$ is the speed of light in vacuum ($v$ in the cavity medium), $Len$ is the length of one edge, and $Dk$ is the dielectric constant of the material filling the cavity.

The phase change of the signal leaking into the cavity during this time is:

$$\Delta \Theta = f_{\text{res}} \times t_{\text{RoundTrip}} = f_{\text{res}} \times \frac{2 \times Len}{c} \sqrt{Dk}$$

When the wave that traveled down and back is exactly in phase with the new wave leaking into the cavity, the two waves add. Each time the wave reflects from the front end, the new reflections are coincident with and add to the old waves. The net wave will build to higher and higher amplitude, limited by the energy leaking into the cavity and the losses of the signal while in the cavity.

The condition for building up the wave amplitudes is that the phase change going down and back be an integral number of cycles. This means going down one way is half a cycle change, or the length of the cavity is half a wavelength long.

The condition for resonance is that the round trip phase change be an integral number, $n$, of cycles, or:

$$n = \Delta \Theta = f_{\text{res}} \times \frac{2 \times Len}{c} \sqrt{Dk}$$

For cavities filled with FR4 type materials, this results in the resonant frequencies being:

$$f_{\text{res}}[\text{GHz}] = \frac{c}{\sqrt{Dk}} \times \frac{n}{2 \times Len} \times \frac{\text{inches}}{\text{ns}} \times \frac{\text{ns}}{2 \times Len} = \frac{n}{3 \times \text{Len}[\text{inch}]} = \frac{n}{7.5 \times \text{Len}[\text{cm}]}$$

If the cavity is 3 inches on a side, its resonant frequency is about 1 GHz. Figure 1 shows the simulated amplitude at the far end of this cavity as the input signal frequency is swept. Note the high amplitudes at multiples of 1 GHz.
In a multilayer IC package, 35 mm on a side, the first resonant frequency for the cavity is about

\[
f_{ws}[\text{GHz}] = \frac{n \cdot 7.5}{3.5 \text{ cm}} = 2.1 \text{GHz}
\]

When the clock frequency of the IC is close to 2 GHz, significant noise can be injected into the cavity between power and ground. This is one reason why return planes are selected as the \(V_{ss}\) layers, so that multiple return vias between \(V_{ss}\) planes can be placed adjacent to signal vias to minimize the possibility of exciting the cavity resonance.

In your applications, be sure to estimate the resonant frequency of all cavities based on their longest dimensions. When clock or data harmonics overlap with the cavity resonant frequencies, you have the potential for long range coupling between any signals that run through the cavity. Be sure to tame this problem before it bites.

**Also see:**

- Bogatin's Rules of Thumb
- Demonstrating enclosure resonance
- Sheet inductance of a cavity: Rule of Thumb #16
—Eric Bogatin is the Dean of Teledyne LeCroy’s Signal Integrity Academy and a well-known SI evangelist. Additional information on this and other signal integrity topics can be found at the Signal Integrity Academy, www.beTheSignal.com.