A Sallen-Key low-pass filter design toolkit

Christopher Paul - December 29, 2018

Do we really need another design tool for Sallen-Key filters? Aren’t there enough [i] [ii] [iii] [iv] [v] already? Obviously, you and I think that there’s something more to say on this topic, or you wouldn’t be reading this. Perhaps the simplest justification for my belief is to point out the differences in the answers, to the same request, from two of these tools. When asked for components in the highest Q section of a 1dB, 7th order, 1kHz, DC gain of 3 Chebyshev filter, one tool offers 220pF and 100nF capacitors with a DC gain of 1, and another 10nF and 31.6(!)nF capacitors with a DC gain of 1.316. These sections exhibit noteworthy differences in noise, component tolerance sensitivities, and op amp gain-bandwidth product (GBP) requirements. It is the goal of this article to derive equations for calculating and managing these parameters and implementing that management in a provided spreadsheet.

Consider the second order topology of Figure 1. Assume for the moment an op amp of infinite GBP. Although it contains six passive components, the transfer function of the section is fully defined by only two parameters: the resonance frequency $F_0$ Hz, and the quality factor Q. Therefore, there are four remaining degrees of freedom in the selection of passive components.

![Figure 1](image_url) A schematic of a second order Sallen-Key low pass filter.

A recent article in EDN [vi] [vii] described how to calculate R1 and R2 given C1, C2, Rf, Rg, Q and $F_0$. It also showed that the sensitivity of the response magnitude, at resonance $F_0$, can be expressed exclusively as a function of Q, C1/C2 and Rf/Rg.[1] These results are assumed here and incorporated into this article’s spreadsheet. The next step is to develop a means for evaluating noise performance. Application notes are available [vii] [viii] which provide an excellent overview of noise analysis.
Figure 2 provides a schematic showing the various noise sources that must be considered in the Figure 1 design. Going forward, we’ll focus on the broadband, white noise aspects of these sources.

Figure 2 A Sallen-Key low pass filter showing all noise sources.

It’s convenient to think in terms of the signals in the figure as being volts and amperes per square root Hertz rather than of volts and amperes. The voltage noise source for an X ohm resistor at 20°C is equal to √(4·k·T·X) = 1.272×10^{-10}·√X. Here k is Boltzman’s constant and T = 293.15°K. And we can simply read the op amp noise source values from their data sheets. The following equations can be written by inspection of the Figure 2 schematic:

\[
\frac{v_a-e_{r1}}{r_1} + (v_a-v_o)·s·c_1 + \frac{v_a+e_{r2}+e_{oo}-v_p}{r_2} = 0
\]

\[
(v_p-e_{oa})·s·c_2 + i_p + \frac{v_p-e_{oo}-v_a-e_{r2}}{r_2} = 0
\]

\[
i_m + \frac{v_m-\epsilon_{rg}}{r_g} + \frac{v_m-v_o-\epsilon_{rf}}{r_f} = 0
\]

\[
vo = A·(v_p-v_m)
\]

These equations can be solved for the portion of each individual noise source that contributes to the total op amp output voltage:
Generally, the individual noise contributions calculated above are uncorrelated with the others, which means that the total output voltage should be equal to the square root of the sum of their squares. However, there is an exception to this. The two noise currents from the bases of an input differential bipolar transistor pair contain components which are correlated with one another (hence the equation for $i_{corr}$) and ones which are not ($i_{m}$ and $I_p$). The correlated parts come from equal splits of the emitter bias current and, if present, base bias cancellation circuit currents. The uncorrelated parts come from the bases of the individual transistors of the pair. These two current types must be handled in computationally different fashions.

The voltages resulting from the uncorrelated portions are among the sum’s squares. However, the difference of the voltages due to the correlated portions must be squared before being added to that sum. If both bases see identical impedances, the correlated currents produce voltages which cancel and do not contribute to the total noise, just as equal DC bias currents seeing equal resistances would yield cancelling results. And so, in the range of frequencies below resonance, matched impedances can offer this beneficial effect. Unfortunately, this is not possible at and above the resonance frequency of our filter due to impedance variations with frequency. Some manufacturers’ datasheets separately call out correlated and uncorrelated currents. For those that don’t, there is a measurement technique [ix] which allows each to be determined.
The op amp gain, $A$, requires some discussion. This is a frequency-dependent parameter equal (to a good approximation in most cases) to $f_{\text{gbw}} / (j \cdot f + f_p)$. Here, $f$ is the frequency in Hz, $f_p$ is a low frequency pole, $f_{\text{gbw}}$ is the GBP, and $j = \sqrt{-1}$. Refer to Figure 3 for a graph of this expression using typical values. Data sheets don’t commonly specify $f_p$, but they do specify $f_{\text{gbw}}$ and $A_{\text{DC}}$, where $A_{\text{DC}}$ is the open loop DC gain. $f_p$ is equal to, and will be replaced with, $f_{\text{gbw}} / A_{\text{DC}}$ in later calculations.

![Figure 3](image-url) A graph of the magnitude of $A = f_{\text{gbw}} / (j \cdot f + f_p)$ for a typical op amp where $f_p = 100$Hz and $f_{\text{gbw}} = 10^7$Hz.

The application of this dimensionless parameter, $A$, to voltage feedback amplifiers is obvious. But Sallen-Key topologies can also employ current feedback amplifiers, whose gains have units of impedance. Fortunately, there is a way to use the equations that have been developed for these devices too [x] (Figure 4). With a little bit of algebra, we can see that for the current feedback amplifier, $A(s) = T(s) / [R_o + R_1 \cdot R_2 / (R_1 + R_2)]$. Op amp datasheets give us $T(s)$ and in some cases $R_o$. When $R_o$ is not supplied, it can be easily determined through simulation or measurement to a reasonable accuracy. This can be done by removing $R_1$ and $R_2$, grounding $V_{\text{IN}}$, and applying a known current $I$ (I is shown in the figure.) $R_o = V/I$, where $V$ is the voltage that arises at the inverting input. Although $R_o$ is typically constant over a broad range of frequencies, it couldn’t hurt to make this measurement near the resonance frequency of the intended filter.

![Figure 4](image-url) Closed loop gains of voltage (left) and current (right) feedback amplifiers, courtesy of Analog Devices.

Next, we consider op amp GBP concerns. Refer to the expression derived for the noise arising from
resistor R1. Its voltage per root Hz noise source in Figure 2 is positioned exactly where a voltage source driving the section would be. From this, we can see that the expression for R1’s contribution to the output noise also provides the voltage transfer function of the entire filter section. Defining \( s_0 = 2 \pi \sqrt{-1} F_0 \), evaluating that expression at resonance and applying a bit of algebra, we get

\[
H(s_0) = \frac{-s_0}{1 + \frac{s_0}{2 \pi f_p} + \frac{f_p}{f_{gbw}} \left( \frac{f_p}{rg} + 1 \right) \left[ \frac{f_p}{rg} + 1 \right] r1 c1 \omega_0 Q + 1}
\]

With an ideal, infinite gain op amp, the magnitude of the denominator is unity. But with finite gain, the denominator’s magnitude grows and that of \( H(s_0) \) falls. We can see that the following inequality constrains the error in the magnitude of \( H(s_0) \) to be less than \( E_{db} \) decibels:

\[
E_{db} = 10 \log \left( \frac{1 + \frac{f_{gbw}}{A_{DC}} K}{1 + (F_0 K)^2} \times 10^{10} \right)
\]

In the above expression, we have replaced \( f_p \) with \( f_{gbw} / A_{DC} \). The equality of the two terms in the square brackets comes from constraining the filter to have the specified Q and \( F_0 \) with an infinite gain op amp. The spreadsheet uses the entry of the parameter \textit{max magnitude response error at resonance} (\( E_{db} \)) on the Design Parameters tab to evaluate the inequality and avoid displaying those portions of curves on the Noise and Sensitivity Graph tab which fail that constraint.

**Spreadsheet features**

The filter section’s, \( F_0 \) and \( Q \), are among the parameters specified on the Design Parameters tab shown in Figure 5. We add to these values the op amp’s voltage and current noises, \( f_{gbw} \) and \( A_{DC} \), the DC open loop gain. Additionally, values for \( C1, C2 \) and \( Rg \) are specified. From these, the spreadsheet generates eight pairs of sensitivity and noise curves on the Noise and Sensitivity Graph tab (see example in Figure 6). Each curve pair covers values of \( Rf/Rg \) from \( 10^{-5} \) to \( 10^1 \) and reflects a value of \( C2 \) equal to \( C1 \) multiplied by \( 10^{-0.5} \), \( 10^{-1} \), \( 10^{-1.5} \), \( 10^{-2} \), \( 10^{-2.5} \) or \( 10^{-3} \). This approach allows consideration of a range of alternative designs. Note that increasing \( C2 \) and reducing \( Rg \) will generally tend to reduce noise by reducing resistor values and therefore the thermal noise they generate, as well as the voltages arising across them due to op amp current noise. Also note that reducing \( Rf/Rg \) and \( C2/C1 \) will typically reduce the sensitivity of a design.

There is a Design Parameters tab parameter entitled, “max magnitude response error at resonance.” If this value is too small for the specified op amp gain and the values of \( Q, F_0, C1, C2 \) and \( Rf/Rg \), certain portions of the Noise and Sensitivity Graph curves will be blanked out. These absent portions are where the error exceeds the specified limit. An op amp’s GBP requirements are generally most critical at resonance, particularly for high Q designs. For this reason, the maximum allowed error is
specified at this frequency.

If you also enter the value of $R_f$, the values of $R_1$ and $R_2$ (green-background cells) which correspond to $Q$, $F_0$, $C_1$, and $C_2$ will be calculated assuming an op amp with infinite GBP. Additionally, a vertical line corresponding to $R_f/R_g$ will be drawn on the Noise and Sensitivity graph. Its intersections with the curves, with the associated value of $C_2$, correspond to this specified filter. That filter is examined on the Noise vs. Frequency Graph tab (Figure 7), whose graph shows the total noise at the op amp output in volts/√Hz along with the contributions of each individual noise source. The individual curves are useful in determining the biggest contributor to noise. If it’s the op amp, choosing a different one can lower the total noise. All curves run from two orders of magnitude below to two above the resonance frequency. The root mean square output voltage noise over this range is also calculated and displayed.

**Spreadsheet examples**

High $Q$ designs provide the most dramatic variations in noise and sensitivity. The highest $Q$ section of a seventh order 1dB ripple Chebyshev has a $Q$ of 10.899. For -1dB at 1kHz with respect to that at DC for the entire filter, this section’s $F_0$ is 996.33Hz.

Asking for a complete filter DC gain of 3V/V, one semiconductor manufacturer’s tool sets $C_1 = 31.6nF$, $C_2 = 10nF$, $R_1 = 59.0k$, $R_2 = 1.37k$, $R_g = 2.49k$ and $R_f = 787$ ohms. The tool assumes an ideal op amp. There are some problems with these component values. First, although the resistors are standard values, $C_1$ is clearly not. Matching it to three-digit accuracy requires the paralleling of two standard capacitors. Since available capacitor values are spaced farther apart than are resistor values, it would have been better to choose standard capacitor values and accept the closest resulting value resistors. Second, substituting these values into the expression for $Q$ given in an application note from the same manufacturer (and also used by this spreadsheet) gives a value for $Q$ of 11.076 rather than 10.899.

The best that the spreadsheet can do to evaluate this tool’s result is to capture the values of $Q$, $F_0$, $C_1$, $C_2$, $R_f$ and $R_g$ it specifies. This is shown in Figure 5. Note the small but distinct differences in the spreadsheet’s computed values of $R_1$ and $R_2$.
for smaller ratios of $R_f / R_g$ because at such values, the filter is unrealizable. This is unrelated to op amp GBP; even higher gains reveal no further portions of the curves. However, if $R_f$ is increased to 1370 ohms so that $R_f / R_g$ is about .55, the sensitivity and noise fall by roughly a factor of three to values which are at local minima— a big improvement from just changing some resistor values. Independent proof of the superior sensitivity of the $R_f = 1370$ ohm saddle point is confirmed by using LTSpice to perform Monte Carlo analyses of filters for which $R_f$ equals 787, 1370 and 24900 ohms. Here the resistor tolerances are 1% and the capacitors’ are 5%.

Figure 6 The curves in the graph above were generated from the parameter values in the yellow background cells in Figure 5, with the exception of that of $R_f$. The interception of the vertical dashed black line with the black curves reflects filters with the ratio of $R_f / R_g$ of Figure 5. The black curves reflect the selected value of $C_2$, while the other colors reflect other values of $C_2$.

Figure 7 The plots in the graphs above labeled with values of $R_f$ (in ohms) are Monte Carlo analyses of filters. Tolerances for the green and blue curves are 1% for resistors and 5% for capacitors. The red “perfect” curves represent filters with 0% tolerance components. The green curves on the left graph reflect $R_f = 787$, while those on the right reflect $R_f = 24900$. The lower sensitivity of the $R_f_{1370}$ ($R_f / R_g = .55$) curve compared to that of the other two is evident, and is in accordance with
These results can be seen in Figure 7. LTspice noise analyses of the same circuits yield resonance frequency values of 1.1E-6, 0.51E-6 and 4.4E-6 Volts per root Hz. All results are in accord with the predictions of Figure 6.

It’s worth looking at what happens when the op amp GBP is reduced to a more realistic (but admittedly somewhat anemic) .5MHz, while tolerating a maximum resonance frequency magnitude error of .1dB. Figure 8 reveals that significant portions of the Figure 6 curves that guaranteed a maximum magnitude error of .1dB with an unrealistically large GBP op amp have disappeared. The tool’s recommended solution at Rf / Rg = .316 is among those which no longer meet that requirement.

The analysis of this filter section wouldn’t be complete without checking the Noise vs. Frequency graph in Figure 9 which uses the original, “ideal” op amp GBP of 1 TeraHertz. As promised, the noise contributions of the op amp to the total output noise of the section are negligible. Beyond providing a general insight, the chief benefit of this graph is to determine whether the use of the selected op amp is limiting the noise performance of the design, which if so could benefit from a lower noise op amp. Of course, the ideal op amp assumed by this manufacturer has been approximated with noise currents and voltage which have negligible effects on output noise.
Figure 9 This graph displays the individual contributions to and the total noise of the filter section described in Figure 5.

Different choices

A second manufacturer’s tool offers different choices. It proposes to use a CMOS op amp with no current noise spec, and so the very small current noise “ideal” values of $10^{-15}$ have been retained. The voltage noise at 1kHz, however, is 10nV / root Hz, the op amp gain-bandwidth is 4MHz and the DC gain is 68 V/mV minimum. C1 is 100nF, C2 is 220pF, R1 and R2 are 34.8K, Rf is 0 and Rg is infinity. Values of 1Ω and 10k have been entered in the spreadsheet for Rf and Rg, because the black curves in Figure 10 stop for Rf / Rg values below $10^{-4}$. The spreadsheet returns slightly different values from the E96 standard 34.8k for R1, because it does not select for standard E-series values. This set of component values reduces sensitivity by a factor of more than 35 times that of the Figure 6 graph with Rf / Rg ≈ .6, but at the price of more than ten times the noise.
A second manufacturer’s tool (intersection of black curves and vertical line) offers a solution with much less sensitivity but much more noise than that of $R_f/R_g \approx 0.55$ in Figure 6.

Of course, these alternatives are not the only choices. It’s worth considering what happens when $C_2$ in the Figure 10 design is changed to 820pF and $R_f$ to 100 ohms ($R_f/R_g = 10^{-2}$.) The result is shown in Figure 11. This is a compromise between the Figures 6 and 10 noises and sensitivities.

*Figure 11* A compromise between the noises and sensitivities of Figures 6 and 10.

It is also instructive to look at the noise and sensitivity curve pairs for a low Q design. *Figure 12* is derived from a $Q = 1.297$, $F_0 = 480.052$Hz section of the earlier-cited Chebyshev filter. Here we
clearly see the noise advantages in choosing a large $C_2$. And as long as $R_f / R_g$ is less than about .1, that choice is optimal for both sensitivity and noise.

Figure 13 The response of the spreadsheet to a request for a physically unrealizable filter.

As we have seen, given an $F_0$ and $Q$ requirement and a few component values, the spreadsheet presents a range of options for the remaining components, calculates the sensitivity and noise at resonance of the resulting second order sections, and displays the contributions of each individual noise source and the total noise over a wide range of frequencies around resonance. For a given response type, there are numerous sources for $F_0$ and $Q$. The semiconductor manufacturers’ design tools are obvious ones. You can also find tables published in many sources [xi] [xii]. Care should be exercised; in some tables, the resonance frequencies of the filter sections are given in radians rather than Hz.

Second order sections deal with two poles, and these are all that is needed to design filters of even order. A different topology is needed for the one “unpaired” section in odd order filters. Typically, this requires an additional op amp to buffer the extra stage. This is often viewed as a reason to avoid odd order filters, since that op amp supports only a single pole of attenuation. However, there is a way to turn that disadvantage into an advantage: combine the single pole with another pole pair to make a one-op amp third order section [xiii] [xiv]. In the absence of a detailed analysis of noise and sensitivity performance of such sections, it is wise to choose one of the lower $Q$ pole pairs for this option. The noise (and sensitivity) of either of these sections can be easily simulated with a Spice tool.

Once the $F_0$ and $Q$ of a section have been selected and entered in the spreadsheet Design Parameters tab, an op amp must be chosen and its GBP, low frequency pole and noise characteristics provided. Finally, component values for $C_1$, $C_2$ and $R_g$ must be supplied. It is helpful to make an initial guess for $R_f$ so that the value of $R_f / R_g$ can be reflected by the black vertical graph line on the Noise and Sensitivity tab. All of these should be standard, commercially available component values. At this point, the curves on that tab can be inspected. It may be that there is no point on one (or interpolated between two) of those curve pairs that provides a satisfactory solution. Lesser sensitivities are generally revealed if the Design Parameter tab’s ratio of $C_1/C_2$ is increased, and lesser noise results occur generally if the product $C_1 \cdot C_2$ is increased. Noise performance might also be improved by viewing the Noise vs. Frequency tab graphs and selecting a different op amp. In all cases, the next step is to identify the least objectionable point available, determine the associated value of $R_f$ and $C_2$, and enter them in the Design tab so that the new selection can be reviewed again on The Noise vs. Frequency tab. You can again determine the biggest single contributor to output noise and hopefully iterate the complete design to an acceptable solution. Choose the nearest available standard values in place of the $R_1$ and $R_2$ reported by the spreadsheet.

The spreadsheet user should be aware that the minimum op amp GBW is usually not specified in the
datasheet. The author is unfortunately unable to recommend appropriate de-ratings of the typical published GBPs. However, private communications with a noted op amp designer for several well-known companies has suggested that 25% could be a good choice. The author welcomes feedback on this topic from knowledgeable readers.

What aspects of filter design does the spreadsheet not address? Most notably, it does not recommend a specific order of connection for multiple sections to minimize signal clipping and/or noise at the filter output. It is hoped that readers will find the article and spreadsheet to be valuable. Comments shall be welcomed.

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[1] The spreadsheet in the prior article employed the parameter $C_{tol}$, the ratio of capacitor to resistor tolerances. In this article, $C_{tol}$ has been fixed to unity. This allows a single graph to display the sensitivities of a wide range of passive component sets satisfying a given $F_0$ and $Q$. For reasonable values (1 to 5) of $C_{tol}$, $C_{tol}$ has been seen to have a small effect on the relative sensitivities of these sets, although it has a larger effect on absolute sensitivities. Since the goal is to make the best choice among those available, the absolute values are not a concern.

References

[i] Active Low-Pass Filter Design and Dimensioning

[ii] Analog Filter Wizard, Analog Devices

[iii] FilterPro, Texas Instruments

[iv] FilterLab Filter Design Software, Microchip Technology

[v] Active Filter Design, Renesas

[vi] Designing second-order Sallen-Key low-pass filters with minimal sensitivity to component tolerances, EDN


[viii] Noise Analysis in Operational Amplifier Circuits, Texas Instruments

[ix] Op amp input current noise, EDN

[x] Current Feedback (CFB) Op Amps, Analog Devices

[xi] Basic Linear Design: Analog Filters, Analog Devices

[xiii] Design second- and third-order Sallen-Key filters with one op amp, EDN

[xiv] Building optimal sensitivity third-order low-pass filters with a single op amp, EDN